ARCH IN SHORT-TERM INTEREST RATES: CASE STUDY USA

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ABSTRACT

We investigate ARCH effects in short term interest rates. Many of the models used to study interest rate dynamics posit a linear function for the conditional mean. Recent work has shown that there are significant non-linearities in the structure of interest rates. We use a neural network model to capture the non-linearities. We find that the evidence for ARCH in short-term interest rates is somewhat overstated. There is some evidence of ARCH effects, but the persistence is not as long as prior estimates have indicated. *JEL Classification: F40*

INTRODUCTION

The volatility of the short rate is a key element in the valuation of interest sensitive contingent claims. Thus considerable attention has been paid to characterizing the data generating process of this rate. The fact that large changes in a time series tend to be followed by large changes and small changes tend to be followed by small changes (volatility persistence) has been noted in the literature since Mandelbrot (1963). There has been an explosion in volatility modeling in the last 25 years since the seminal papers by Engle (1982) and Bollerslev (1986) who introduced the AutoRegressive Conditional Heteroskedasticity (ARCH, henceforth), and Generalized AutoRegressive Conditional Heteroskedasticity (GARCH, henceforth) models respectively.

In the volumes of work since then, there exists significant evidence that volatility also appears to contain long-memory characteristics, the pattern of which seems to be a rapid decrease at first, and then a hyperbolic decrease for many periods. This has led to the development of fractionally integrated GARCH model (FIGARCH) of Baillie, Bollerslev and Mikkelsen (1996) and long memory stochastic volatility models (Breidt, Crato and DeLima 1998). There has also been a lot of recent interest in non-linear modeling of the conditional mean and variance (Ait-Sahalia 1996, Tauchen 1997, etc). Non-linearities in the underlying time series may lead to over rejection of the “no ARCH” hypothesis and may be the cause of some of the “evidence” for long-memory.

We use the method described by Blake and Kapetanios (2007) to apply non-parametric modeling using neural networks to capture possible non-linearities in the data generating process. Having modeled the conditional mean, we can then apply any conventional tests for ARCH to investigate the remaining conditional heteroskedasticity. These models are intended to capture the persistence in volatility, which seems to be a salient feature in financial markets.

The paper proceeds as follows. Section II shows the nature of the problem. Section III introduces the Neural Network method of modeling flexible functional forms, followed by the estimation and results in Section IV. Section V concludes the study.

ARCH AND NON-LINEARITIES

Volatility persistence has long been known to be a feature of financial time series. Since Engle's seminal paper (Engle1982) there have been a proliferation of models in the ARCH/GARCH line, and today it is a standard feature of any time series and financial software. The estimates from these models tend to indicate that volatility is very persistent. Indeed the GARCH model and long-memory models were proposed to explicitly take into account the very long lag structure in volatility modeling.

Most of the models used assume a linear function for the conditional mean of the time series. Recent work indicates that non-linear models may be more appropriate. Chan, et al (1992), Brenner, Harjes, and Kroner (1996), Ait-Sahalia (1996), Conley et *al* (1997), Tauchen (1997), provide empirical evidence casting doubt on the linearity of single factor continuous models for the interest rate. The Markov regime-switching models proposed by Hamilton (1988, 1989, 1990) and applied to interest rates by Austin (2002), Bansal and Zhou (2002), and Gray (1996) were created to model non-linearity in discrete time.

The basic structure of the ARCH model is given by:

$$y_t = \mu_t(y_{t-1}, y_{t-2}, y_{t-3},\ldots;\beta) + \varepsilon_t \tag{1}$$

$$h_t = \alpha_0 + \sum_{i=1}^{k} \alpha_i \varepsilon_{t-i}^2 \tag{2}$$

where h_t is the conditional volatility. These models typically use a linear specification for the conditional mean. However as is well understood, if the conditional mean is actually non-linear this may lead to over-rejection of the null hypothesis of no ARCH. Blake and Kapetanios (2007) show that non-linearities in the conditional mean can significantly affect tests for ARCH since the mis-specified model can lead to squared residuals that are correlated even in cases where there is no ARCH.

Since the volatility is a key component of many pricing models, this may have serious consequences for the accuracy of these results. Following Blake and Kapetanios (2007), we use neural networks to estimate the non-linearities in the conditional mean. This produces residuals which converge in probability to the true error terms and can thus be used to test for the presence of ARCH. Our results are twofold. First, we find that although there is strong evidence for ARCH effects in the series, the lag lengths are much shorter than previously estimated. (1) Second, we also show the type of mis-specification inherent in linear-models, which suggests the manner in which these models should be modified.

NEURAL NETWORKS AND THE RADIAL BASED FUNCTIONS (RBF)

An Artificial Neural Network (ANN, henceforth) is a non-linear statistical computational tool based on adaptive biological neural networks. Radial Based Function (RBF, henceforth) networks are ANNs that can approximate any function $\Re^N \to \Re$ defined on a compact subset of $\Re^N$ with any degree of precision (Campbell, Lo, and MacKinlay 1997). We use the RBF network to approximate the conditional mean function of the transitional density of the short rate. We concentrate on the following univariate model for the short rate:

$$r_t = f(r_{t-1};\beta) + \varepsilon_t \tag{3}$$

where r_t is the short term interest rate at time t, $f(\cdot|\cdot)$ is an unknown, continuous, function and ε_t is a random variable with mean 0 and conditional variance given by:

$$h_t = \alpha_0 + \sum_{i=1}^{k} \alpha_i \varepsilon_{t-i}^2 \tag{4}$$

Given the universal approximation property, we know that we can use an RBF network to write f as:

$$f(r_t;\beta) = b_0 + b_1 r_t + \sum_{i=1}^{m} a_i g((d_{0,i} + d_i r_t)) \tag{5}$$

where g is given by:

$$g(r_t;d_j,\gamma) = e^{-(\frac{r_t - d_j}{\tau})^2} \tag{6}$$

and d_j is the j^{th} center. The function has a maximum of 1 when r_t coincides with the center, and goes to 0 as r_t goes further away from the center. The rate at which the function decreases is determined by τ. The network is defined by the choice of centers and radius τ. If τ and the centers $\{d_j\}_{j=1}^m$ are known then the RBF is easy to estimate using least squares.

We follow Blake and Kapetanios (2007) by choosing τ, and $\{d_j\}_{j=1}^m$ by a data dependent method independently of the RBF. τ is chosen to be the variance of the data (or alternatively 1 for normalized data), and we allow there to be "T" potential centers. Each of the centers is an observation drawn from the data. The centers are then ranked by their ability to reduce the unexplained variance when entered individually. Then we successively add the ranked centers until we minimize an information criterion. Having chosen the centers and the radius, the neural network becomes linear in a_i. After fitting the network, the residuals can be used to test for the presence of ARCH, and we estimate the model given above.

ESTIMATION

We perform the tests on weekly 3-month interest rates for the U.S. The rates are from the 90 day treasury bill from 1973.06.07 to 2007.01.18, collected on Thursdays. If Thursday data is not available, we use Wednesday's numbers. The data are provided by the Federal Reserve Bank of St. Louis, and is commonly used as a proxy for the short term risk-free rate.

Figure 1

3-month T-bill Rate (United States)

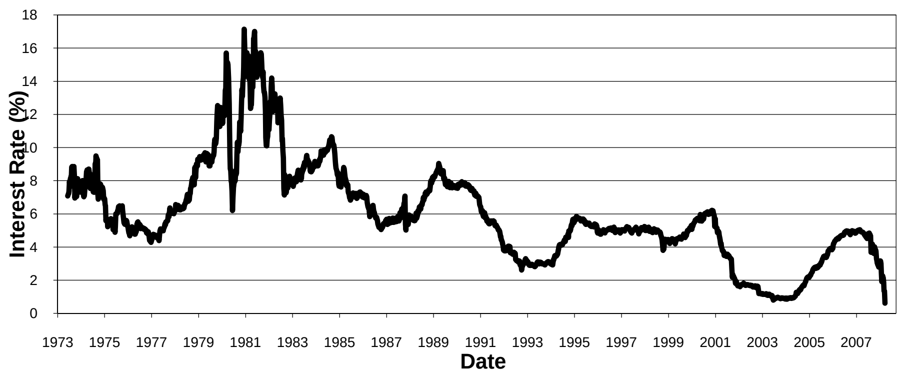
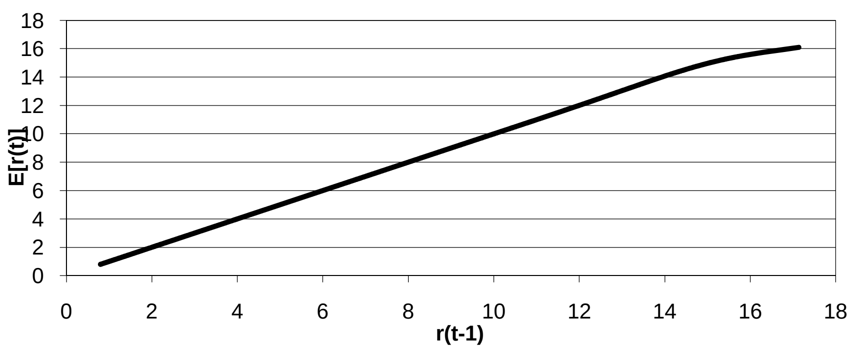


Figure 2

Conditional mean function E[r(t)] = f(r(t-1))



The initial estimates from the linear model are given in table 1. Using the AIC information criterion we find that the US series show that a shock has consequences for volatility for a very long time. We then re-estimate the model using the Radial based neural network (which we refer to as the non-linear model). The results are also given in table 1.

The series shows evidence of ARCH effects, but when we estimate its structure, we find that the lag length to be much shorter, and fewer overall number of parameters than the linear model.

TABLE 1
AKAIKE INFORMATION CRITERIA (AIC)

ARCH lag length	US Linear	US non-linear
α_0	-6.4206	-6.5202
α_1	-6.6404	-6.8215
α_2	-6.6917	-6.8350
α_3	-6.7502	-6.8612
α_4	-6.7619	-6.8627
α_5	-6.7649	**-6.8702**
α_6	-6.7716	-6.8701
α_7	-6.7829	-6.8692
α_8	-6.8135	-6.8682
α_9	-6.8155	-6.8672
α_{10}	-6.8196	-6.8690
α_{11}	**-6.8208**	-6.8142
α_{12}	-6.8197	-6.8117
α_{13}	-6.8189	-6.8037
α_{14}	-6.8177	-6.7521
α_{15}	-6.8185	-6.7040

Table 1: The AIC information criteria for the models with linear conditional mean and non-linear conditional mean at varying ARCH lag lengths. α_i represents a model with i lags in the conditional variance.

Notes:

1) This note is part of a more comprehensive project where we apply this estimation procedure across a larger set of countries. Our results hold consistently over all the time series examined. Results available upon request.
2) We are currently examining interest rates for the United Kingdom, Canada, Japan, and Europe. Preliminary findings show similar results to the US case.

Looking at the conditional mean function (figure 2) also gives us some insight into the structure of the interest rate process. We find that over a certain range of the data (up to about 14) the function is close to linear and there is almost no evidence of mean reversion. However at the high end of the range, the function becomes non-linear and the evidence for mean reversion becomes very strong indeed. This may help to explain some of the controversy over the existence of mean reversion in interest rates, and it also fits in with the findings of Ait-Sahalia (1996) and Gray (1996).

CONCLUSION

We find that for short term US interest rates, there is evidence of both ARCH effects and non-linearity in the conditional mean. We find that with the conditional mean, the volatility persistence is much shorter than is typically found in estimation of long-term volatility effects. Further study is needed to determine whether these characteristics are unique to the US, or are consistent with other country data.[2] This could also have consequences for the pricing of bonds and options when non-linearities are present.

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