A THRESHOLD COINTEGRATION TEST OF THE FISHER HYPOTHESIS: CASE STUDY OF 5 EUROPEAN NATIONS

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ABSTRACT

A perennial favorite among monetary and financial economists is the superneutrality hypothesis, conceptualizing the relationship between nominal interest rates (r_t) and inflation (π_t). The empirical evidence is far from unanimous. This failure is contributed to the fact that previous studies have ignored the stochastic features of these variables, i.e., their inherent nonlinearities, which have resulted in only a partial adjustment (temporal co-movement) between the variables.

To model the true dynamics of "r_t" and "π_t", now, we have an appropriate and powerful metric in the form of the "threshold cointegration" methodology. We propose to apply it to revisit this as of yet unsettled question, for a set of five European countries namely, Belgium, France, Germany, Italy and Sweden. We find evidence in favor of the presence of linear cointegration for all countries, but threshold cointegration for Belgium and Germany, and mixed evidence for France. Error correction results are given for Belgium and Germany and these indicate a partial Fisher effect with the cointegrating vector being greater than one. This is possibly due to tax effects.

INTRODUCTION

A perennial favorite research topic among monetary and financial economists is the age old "Fisher equation," which conceptualizes the degree (and extent) to which nominal interest rates incorporate changes in expected inflation, thus keeping the real interest rate unaffected. The validity or otherwise of this hypothesis has fundamental ramifications for both theory and policy. As an example, this hypothesis upholds the super-neutrality of money and justifies using nominal interest rates as a good predictor of future inflation.

Two things seem to stand out in the literature. One is the conflicting results regarding its existence / non-existence in an absolute sense. Here for example we have Mishkin (1992) which examines the hypothesis for postwar USA, and finds evidence in support (and against) it over different periods of time. Secondly, when nominal interest and inflation do co-move, it's a case of partial co-movement and not a one-to-one correspondence in the long run. One reason forwarded for this "on again-off again" relationship between the two variables is their inherent nonlinearity, resulting only in partial adjustment when and only when their difference reaches some threshold. This has been examined lately by Evans and Lewis (1995), Garcia and Perron (1996) and Bierens (2000) to name a few.

Here we apply the recently available Hansen-Seo (2002) test of bivariate threshold cointegration, which through Monte Carlo simulation has been proven to be a very powerful test. Its strongest feature is its ability to model the true dynamics of the interest rate and inflation series, where proactive policy results in "mean reversion" only when the difference between the variables reach some threshold. It is applied it to the case of five major European economies namely Belgium, France, Germany, Italy and Sweden, with monthly data on the CPI (for the inflation rate) and the 3 month treasury certificate rates (for the short term interest rate) ranging from 1960 to 2004. They were chosen because they are all highly industrialized open market economies and major players in the European Economic Community.[1]

In the next section we carry out a brief review of the literature on this topic, followed by a brief description of the data set. The following section gives a brief description of the Hansen-Seo (2002) test and results followed by our concluding remarks.

LITERATURE REVIEW

The concept originated in Fisher (1896) and was extended in 1930. According to Professor Fisher nominal interest responds to the changing inflation rate by smaller amounts and with a time lag. The justification is the presence of 'money illusion,' which implies the inability of people to distinguish between nominal and real variables, at least in the short run. A practical manifestation of the presence of money illusion would be that lending institutions would not fully transmit their expected change in the inflation rate to the nominal interest rate, even if they do correctly estimate the expected inflation(r_t^e). The concern with testing the Fisher hypothesis is that it is a long run phenomenon and more and more evidence is coming to the fore that economic / financial variables undergo structural changes over time say due to policy changes and / or business cycle fluctuations. Granville and Mallick (2004) find evidence in favor of the Fisher effect in the U.K. after examining data for 1900 – 2000. Carneiro, Divino, and Rocha (2002) find evidence in favor of the Fisher effect in Argentina and Brazil, but not Mexico, through an examination of monthly data for the period 1980 – 1997. Sun and Phillips (2004) find little support for the Fisher hypothesis in the United States. But the early empirical studies using linear cointegration, have not included the presence of regime shifts.

The rationale provided for the failure of the Fisher hypothesis runs the gamut of logic from money illusion to the negative effect of inflation on money demand to the negative correlation between inflation and real interest rates. Madsen (2005) states that the rejection of the Fisher equation is often due to the omission of supply variables from the Fisher equation as this will lead to an upward bias for the coefficient of expected inflation and make it easier to reject the Fisher hypothesis. This study proposes a technical / econometric reason for the failure of the hypothesis, in terms of the stochastic properties of the variables under consideration. It is also propounded as the reason for the partial acceptance of the Fisher hypothesis, i.e., why it held over some periods and not over others.

To model the true dynamics of interest rate and inflation over time, we use the "threshold cointegration" model which is a recent contribution in economics. It was first used in the biological and physical sciences, to model systems whose behavior changes once they reach some "saturation" point. In the field of international finance, Davutyan and Pippenger (1990) and Balke and Fomby (1997)

amongst others, have used it to examine the character of the purchasing power parity hypothesis. They contend that due to the presence of transaction costs (which are lump sum costs) agents do not adjust continuously, but rather discretely once the price differential is greater than the transaction cost. Now this is a step function in the sense of zero market activity before the bound or threshold has been reached, and complete action once it has been crossed. In the cointegration sense, the error correction mechanism is inactive inside the boundary, but kicks in once the threshold has been crossed. Thus the equilibrium error is modeled as a threshold autoregression in the sense of mean reversion only from outside the band, i.e. the variables are nonstationary (have unit roots) inside the band.

In the context of the Fisher correlation hypothesis, both nominal interest rate and inflation are nonstationary (and do not trend together under normal circumstances), but once the limits of social / political comfort are reached, proactive policy (for example contractionary / expansionary monetary policy) will kick in and the variables will become "mean reverting," and hence start behaving like stationary entities. Their actual behavior depends on the width of the "tolerance band," say for example the differential between the higher (H) and lower (L) bounds as (T_H-T_L). Mean reversion or tendency towards forming a cointegrating vector will be evident only at the limits and / or outside the band. Conventional estimates of this long run relationship has always assumed a "constant difference" and thus resulted in biased estimates. Million (2004) uses threshold cointegration to examine the Fisher hypothesis in the United States, and finds that "the policy maker….changes his behavior depending on the level of inflation: whenever the inflation rate decreases below a level of tolerable inflation the policy maker is more reluctant to conduct an active policy…….However, in a context of increasing inflation….the monetary authorities change nominal interest rates so that inflation rates will go back to acceptable levels." Adusei and Kunst (2002) find evidence in favor of threshold cointegration between long rates and inflation but not for short rates and inflation.

Given the mixed evidence in favor of the Fisher hypothesis, a re-examination is warranted. Furthermore, since there is no theoretical reason for imposing linearity on the hypothesis, a testing procedure that encompasses both linearity and non-linearity would yield the most robust results. The Hansen-Seo approach allows us to do this, and therefore we feel our results on the Fisher hypothesis will be robust and will contribute to the literature in this field.

DATA DESCRIPTION

The data is from the OECD data set, available from the Estima Corporation. All of the data is monthly. The inflation proxy is the CPI (all items, nonseasonalized) and the short term interest proxy is the treasury certificate (3 month rate). The time period is Belgium [1960(1) – 1998(12)], France [1970(1) – 1998(12)], Germany [1960(1) – 1998(12)], Italy [1978(1) – 1998(12)] and Sweden [1982(1) – 2004(04)].

HANSEN – SEO THRESHOLD COINTEGRATION MODEL

Threshold cointegration introduced by Balke and Fomby (henceforth, BF1997) combines nonlinearity and cointegration by allowing for nonlinear adjustments over the long run. It begins by testing for the presence of a bound or threshold where the null hypothesis is of linearity. Hansen –Seo (2002) extend the

BF and the Lo and Zivot (2001) test to examine for the presence of an unknown cointegrating vector where they test the term structure model of interest rates, which suggests that the one period bond rate (r_t) and the multi-period bond rate (R_t) should be cointegrated with a unit cointegrating vector.

The basic model is:

$$\Delta x_t = A \, X_{t-1}(\beta) + u_t \tag{1}$$

where X_t is a p-dimensional I(1) series with one (p x 1) cointegrating vector (β), and u_t is assumed to a vector martingale difference sequence with finite covariance matrix $\Sigma = E(u_t u_t^{'})$. The threshold cointegration model takes the form:

$$\Delta x_t = \begin{cases} A_1^{'} X_{t-1}(\beta) + u_t \;\; if \; w_{t-1}(\beta) \leq \gamma, \\ A_2^{'} X_{t-1}(\beta) + u_t \;\;\; if \; w_{t-1}(\beta) > \gamma \end{cases} \tag{2}$$

where A_1 and A_2 are the coefficient matrices in the two regimes, with $A_1=A_2$ when a threshold does not exist, γ is the threshold parameter and $w_t(\beta) =\beta'x_t$ is the I(0) error-correction term. Hansen and Seo undertake a grid search based on a prior linear and consistent estimate of the cointegrating vector β. The search is conducted for the values of [β, γ] over the range [β_L and β_U] and [γ_L and γ_U]. The threshold test is for H_o (null hypothesis of linear cointegration) versus H_1 (alternative hypothesis of threshold cointegration). Under the null hypothesis of no threshold equation 2 reduces to equation 1 ($A_1 = A_2$). A Lagrange multiplier (LM) test for threshold cointegration is applied and the p-values are calculated. Simulation evidence is provided to justify the size and the power of the tests.

The Fisher hypothesis requires nominal interest rate (r_t) to be cointegrated with the inflation rate (π_t) with an unit cointegrating vector. In order to test for this, equation (1) is estimated in the following form

$$\begin{pmatrix} \Delta r_t \\ \Delta \pi_t \end{pmatrix} = \mu + \alpha w_{t-1} + \Gamma \begin{pmatrix} \Delta r_{t-1} \\ \Delta \pi_{t-1} \end{pmatrix} + u_t \tag{3}$$

with $w_{t-1} = r_{t-1} - \beta\pi_{t-1}$ being the error correction term. When $\beta=1$, the error correction term is the traditional Fisher equation. Any other value of β would imply a partial Fisher effect.

First we run the conventional Engle-Granger (1987) test of bivariate cointegration (to reject the null hypothesis of no cointegration).[2] Then based on these results we examine for threshold cointegration between the nominal interest rate and the inflation rate.

TABLE 1
NOMINAL INTEREST RATES AND INFLATION: TESTS
FOR THRESHOLD COINTEGRATION

	Bivariate		Univariate	
	β=1	β estimated	β=1	β estimated
Belgium	0.0894	0.015	0.0308	0.0058
France	0.0294	0.0324	0.0368	0.3254
Germany	0.013	0.0238	0.0130	0.0000
Italy	0.895	0.8734	0.5686	0.1266
Sweden	0.6266	0.1818	0.6690	0.0094

Note:
This table gives the p-values for tests of threshold cointegration, which
are calculated by a parametric bootstrap, with 5000 replications. The
tests used are the SupLM test for estimated β (the cointegating vector is
estimated, and SupLM^0 for β=1 (the cointegrating vector is estimated).
Bivariate refers to the cointegration tests described in Hansen-Seo (2002).
Univariate refers to the tests described in Balke and Fomby (1997).

For all countries, the test rejects the null hypothesis of no cointegration. In
testing for threshold cointegration, we estimate equation 3 and then use the SupLM
test (estimated cointegration vector) and SupLM^0 test (cointegration coefficient = 1),
with p-values calculated by bootstrap. For comparison, we also calculated the
univariate Hansen (1996) threshold autoregression (TAR) test to the error correction
term. The multivariate tests point to the presence of threshold cointegration for
Belgium, France and Germany. This is not surprising since the ADF test indicated
the presence of cointegration for all three countries. The univariate tests also indicate
the presence of threshold cointegration for Belgium and Germany (at the 5% level)
and mixed evidence for France (the test is significant for the case where β is
constrained to be equal to 1, but is insignificant for the estimated β). For Italy all tests
indicate the absence of threshold cointegration and for Sweden there is mixed
evidence (the test is insignificant for the case where β is constrained to be equal to 1,
but is significant for the estimated β). We report the parameter estimates from our
estimation of equation 3 for Belgium and Germany in Case 1 and 2, since we have the
strongest evidence in favor of threshold cointegration for these two countries.

Case 1:
Let us consider the parameter estimates for Belgium first. The estimated
threshold (γ in equation 2) is -16.54. Thus, the first regime will occur when the error
correction term $w_t \leq$ -16.5484, or $r_t \leq 1.97\pi_t$ – 16.54, where r_t is the nominal interest
rate and π_t is the inflation rate. The first regime has approximately 7 percent of the
observations (and therefore can be called the unusual regime) and the second regime,
which occurs when the $r_t > 1.97\pi_t$ – 16.54, has about 93 percent of the observations
and can be called the typical regime (using terms proposed by Hansen-Seo, 2002).

$$\Delta r_t = \begin{cases} -0.7834 - 0.0377 w_{t-1} + 0.3348 \Delta r_{t-1} - 0.0108 \Delta \pi_{t-1} + u_{1t}, & w_{t-1} \leq -16.5484 \\ (0.3834) \quad (0.017) \quad\;\; (0.1227) \quad\quad (0.0165) \\[2mm] 0.0037 - 0.0099 w_{t-1} + 0.2578 \Delta r_{t-1} + 0.0018 \Delta \pi_{t-1} + u_{1t}, & w_{t-1} > -16.5484 \\ (0.0256) \quad (0.0042) \quad\;\; (0.0918) \quad\quad (0.0079) \end{cases}$$

$$\Delta \pi_t = \begin{cases} 0.4950 + 0.0595 w_{t-1} - 0.6595 \Delta r_{t-1} - 0.6851 \Delta \pi_{t-1} + u_{2t}, w_{t-1} \leq -16.5484 \\ (4.9184) \quad (0.2194) \quad\;\; (0.9699) \quad\quad (0.1197) \\[2mm] 0.3337 + 0.3439 w_{t-1} - 0.5247 \Delta r_{t-1} - 0.0508 \Delta \pi_{t-1} + u_{2t}, w_{t-1} > -16.5484 \\ (0.1903) \quad (0.0339) \quad\;\; (0.4503) \quad\quad (0.0553) \end{cases}$$

(Numbers in parenthesis are standard errors)

In the typical regime ($w_t > -16.5484$), both Δr_t and $\Delta \pi_t$ have significant error correction terms, which is negative for Δr_t but positive for $\Delta \pi_t$. This implies that both interest rates and inflation respond to deviations from the long run equilibrium. This is not surprising because when the difference between the nominal interest rate and inflation becomes too high, we would expect the interest rate to decrease back to the equilibrium level, as indicated by the negative coefficient on the error correction term. Also, if the difference between nominal interest rate and inflation rate is high, the inflation rate would increase back to the equilibrium level, as indicated by the positive coefficient on the error correction term. In the unusual regime the effects are significant and negative for Δr_t but insignificant for $\Delta \pi_t$. The lagged value of interest rates has an impact on interest rates in both regimes, implying that interest rates are persistent (market is not weak form efficient), but lagged inflation does not have an impact on interest rates. On the other hand, lagged inflation has an impact on inflation only in the unusual regime, implying that inflation, by and large is persistent.

The estimated cointegrating relation is $w_t = r_t - 1.97 \pi_t$, which seems to be significantly different from a unit coefficient. This implies that a 1 percent increase in inflation will yield a 1.97 percent increase in nominal interest rate. One possible reason for the absence of a 1 to 1 relationship between inflation and nominal interest rate could be taxes. Since we use before tax interest rates, the use of after tax rates could lead to a full Fisher effect, as obtained in Crowder and Hoffman (1996).

Case 2:

For Germany, the estimated threshold is 2.25. Thus, the first regime will occur when the $r_t \leq 1.87 \pi_t + 2.25$. The first regime has approximately 58 percent of the observations and the second regime has about 42 percent of the observations. We do not classify either regime as typical or unusual since the percentage of observations is relatively close. The parameter estimates for Germany are

$$\Delta r_t = \begin{cases} -0.0389 - 0.0136 w_{t-1} + 0.3240 \Delta r_{t-1} - 0.0133 \Delta \pi_{t-1} + u_{1t}, & w_{t-1} \leq 2.25 \\ (0.0288) \quad (0.0061) \quad\;\; (0.0869) \quad\quad (0.0076) \\[2mm] 0.0331 + 0.0066 w_{t-1} + 0.4319 \Delta r_{t-1} + 0.0388 \Delta \pi_{t-1} + u_{1t}, & w_{t-1} > 2.25 \\ (0.0615) \quad (0.0091) \quad\;\; (0.1014) \quad\quad (0.0108) \end{cases}$$

$$\Delta\pi_t = \begin{cases} \underset{(0.2754)}{0.4770} + \underset{(0.0414)}{0.4483}w_{t-1} + \underset{(0.3493)}{0.2991}\Delta r_{t-1} - \underset{(0.0524)}{0.0114}\Delta\pi_{t-1} + u_{2t}, & w_{t-1} \leq 2.25 \\[2em] \underset{(0.5017)}{0.5369} + \underset{(0.0830)}{0.2828}w_{t-1} + \underset{(0.9084)}{0.0794}\Delta r_{t-1} - \underset{(0.0807)}{0.0293}\Delta\pi_{t-1} + u_{2t}, & w_{t-1} > 2.25 \end{cases}$$

(Numbers in parenthesis are standard errors)

For the first regime, the error correction terms are significant and negative for Δr_t and significant and positive for $\Delta\pi_t$, implying that both interest rates and inflation respond to deviations from the long run equilibrium. For the second regime, the error correction term is insignificant for Δr_t and significant and positive for $\Delta\pi_t$. As we explained for Belgium above, we would expect the equation for Δr_t to have a negative error correction term since a large difference between nominal interest rates and inflation would lead to the nominal rate decreasing such that the equilibrium relation is reached, and the equation for $\Delta\pi_t$ should have a positive sign as a large difference between nominal interest rate and inflation would result in an increase in the inflation rate such that equilibrium is again reached. For Germany the cointegrating relation is $w_t = r_t - 1.87\pi_t$, and therefore β seems to be significantly different from 1. As explained above for Belgium, this could also be due to the existence of a tax effect. Lagged interest rates do have an impact on interest rates and lagged inflation also has an impact on interest rates. On the other hand, lagged interest rates do not have an impact on inflation, and lagged inflation has an impact on inflation only in the first regime.

A full Fisher effect would imply a unit cointegration vector. A cointegration vector greater than 1, as we have obtained above, could be due to the existence of a tax effect. Crowder and Hoffman 1996) using U.S. data get a before tax cointegration vector of 1.34 and an after tax cointegration vector close to 1. Since Europe, in general, has higher marginal tax rates compared to the United States, the higher values of the cointegration vector which we have obtained for Belgium (1.97) and Germany (1.87) could be consistent with an after tax full Fisher effect. That analysis has not been carried out in this paper since we do not have information on marginal tax rates in Europe.

CONCLUSION

Conventional (linear) approaches to estimating long run relationships (as the one between inflation and interest rates) are generally mis-specified, since these coefficients reflect average changes across different regimes / structural shifts and hence are biased by themselves. In reality in today's world, due to the presence of active (and in many instances pro-active) monetary / fiscal / government policy, there are prescribed (explicit or implicit) thresholds, which are politically acceptable limits or bounds which when reached and / or crossed, will result in a regime shift. The longer the sample period, the larger the probability of multiple regimes / thresholds, and hence the more biased are the estimated linear coefficients. Here threshold cointegration becomes relevant since it allows for nonlinearities in the underlying data generating process (DGP) of the variables, both in structure and co-movement. Thus the results present a more realistic picture of the real world. Case in point here is the results of Mishkin (1992), which upholds the Fisher hypothesis strongly for the

period between November 1979 and October 1982 (when inflation reached all time post war highs and thus reached and / or breached the upper threshold), but less so otherwise. This can be theoretically justified using the logic of threshold cointegration. It also validates the Phylaktis and Blake (1993) study which reports evidence upholding Fisher for high inflation economies, as opposed to little favorable evidence for the low inflation economies.

Lagged interest rates seem to have a significant impact on interest rates for both Belgium and Germany. This is possibly due to the constraints placed on monetary policy by the requirements to keep exchange rates within the bands of the Exchange Rate Mechanism. Lagged inflation had an impact on interest rates only for Germany but not for Belgium. The German Central Bank always had an anti-inflationary bias to its monetary policy, often to the exclusion of other goals. This approach was not always followed by other central banks in Europe. Lagged inflation had an impact on inflation only in one regime for both Germany and Belgium. Since the objective of monetary policy was to keep exchange rates stable, inflation was only a secondary objective, if it was indeed an objective.

One significant result we have is the absence of a full Fisher effect, since the cointegrating coefficient we have obtained for both Belgium and Germany is significantly greater than one. One explanation for this is the existence of tax effects, and the use of after tax nominal interest rates could lead to a full Fisher effect, as in Crowder and Hoffman (1996).

ENDNOTES

[1] The fact that the relevant data was readily available in case of these economies was also a consideration, though not the overwhelming reason to choose this set.
[2] All empirical results were estimated using GAUSS. The GAUSS programs used for the estimations are available on the website of Bruce Hansen.

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