INPUT DECISIONS OF COMPETITIVE FIRMS UNDER UNCERTAINTY: SOME COMPARATIVE ANALYSES USING RISK-RETURN MODELS

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ABSTRACT

The theory of the firm under uncertainty has been usually studied using the expected utility approach. This paper applies various mean-risk models to the input decision making of a competitive firm facing product price uncertainty. The risk formulation used in this paper includes a prespecified target level of profit, similar to those discussed in Fishburn (1977). Furthermore, the measure of risk-aversion used in this model indicates the relative weight given by the agent to the risk component of the mean-risk objective function. The expected utility results require decreasing absolute risk aversion as a sufficient condition. But the corresponding sufficient condition in the risk-return model of this paper is the ratio of wages to marginal productivity of labor that is at least as large as the sum of average input cost and average target level of profit. Increases in input prices and firm's risk aversion reduce input demands resulting in lower output.

INTRODUCTION

This topic is of particular importance because business firms are exposed to more intense competition causing them to lose their product-pricing power to increase total profit. As a result, their product-demand conditions are more uncertain making product prices highly unpredictable. Consequently, firms need to allocate and utilize inputs in the production process to minimize production cost to enhance competitiveness. The topic is also interesting and merits an indepth theoretical analysis because firms now face more uncertainties because of i) rising product price competitiveness ii) unexpected input supply disruptions, iii) higher competition in input procurements and pricing, iv) uncertain technological progress that unpredictably alters market dynamics and v) fragmentation in the production process (outsourcing). Because of these factors, firms need to pay closer attention to efficient input allocations and utilizations thereof to reduce both variable and fixed costs. In short, to be competitive, firms have to be cost-efficient in an uncertain market environment. This paper thus analyzes efficient input decisions by firms under uncertainty.

For example, GM and Ford have been facing steep price competition from foreign auto-suppliers like Japanese Toyota and Honda. To reduce cost, GM and Ford outsource various components. They are also forced to downsize their U.S. workforce and reduce capital expenditures. Colossal losses and shrinking market-share have raised their risk aversion. As a result, the hiring of auto workers and capital outlays are declining which reduce auto production by GM and Ford in the U.S. This phenomenon conjecturally accords well with the major findings of this paper.

Risk-return models have been used extensively in portfolio theory. The theory of the firm under uncertainty has not been analyzed with the risk-return approach except for a limited number of works, such as, the mean-standard deviation model of Hawawini (1978). The expected utility approach has become popular in this field of study notwithstanding the intuitive appeal of the risk-return dichotomy in the business world. This paper applies various risk-return models to the input decision making of a competitive firm facing uncertain product prices. The results of the risk-return models are compared to those of the expected utility literature. The risk-return approach requires moderate restrictions on the cost function of the firm. The expected utility approach requires a more stringent assumption of decreasing absolute risk aversion in order to obtain deterministic comparative static results. Our attempt is to explore whether different results arise under risk-return vs. expected utility approaches.

The remainder of the paper is organized as follows. Section II presents literature review. Section III is about the purpose of the paper and basic assumptions of the models. Section IV discusses mean-general risk model. Section V is about mean-standard deviation model. Finally, section VI offers conclusions.

LITERATURE REVIEW

Mills (1959) studies a monopolist under uncertainty and is one of the earliest works introducing uncertainty in microeconomics. But Mills' work suffers from serious limitations, such as, assumption of risk neutrality. Zabel (1970) is a generalization of Mills' article in some respects. Zabel uses multiplicative form of uncertainty instead of additive separability of uncertainty. Sandmo (1971) and Baron (1970) deal with a competitive firm facing an uncertain price of its product. These two articles are complementary to each other. The marginal impact of changing the distribution of price is studied by Sandmo but not by Baron, while Baron studies the effect of an increase in risk aversion which Sandmo ignores. Sandmo finds that the overall-impact of uncertainty is to reduce output assuming that the marginal cost is rising. However, Sandmo himself could not determine the sign of the marginal impact of uncertainty (for example the effect of a mean-preserving spread). But this problem was later taken up by Ishi (1977) who showed that nondecreasing absolute risk aversion is a sufficient condition for the marginal impact to be in the same direction as the overall impact.

Baron proves that "…optimal output is a nondecreasing function of the firm's index (Arrow-Pratt measure) of risk aversion." Moreover, an increase in fixed cost decreases output for decreasing absolute risk aversion. Baron also concludes that if risk aversion is prevalent, as seems reasonable, prices are higher and outputs are lower than if firms were indifferent to risk. Finally, Baron finds that under uncertainty it is possible for the short run supply function of the risk averse firm to have a negative slope. This is a result which cannot occur in deterministic microeconomic theory. Baron (1971) demonstrates that for an imperfectly competitive firm under uncertainty the strategies of offering a quantity or changing a fixed price yield different results. Leland (1972) considers three alternatives behavioral modes under uncertainty, and claims that the result of Baron (1970) and

Sandmo (1971) are special cases of his more general results. Lim (1980) addresses the question of ranking these behavioral modes under risk neutrality.

Batra and Ullah (1974) follow Sandmo heavily except that they use a production function and adopt an input approach instead of an output approach. As shown by Hartman (1975,1976), the Batra and Ullah paper suffers from a partial approach in deriving their conclusions about the overall impact of uncertainty on input demands considering both inputs simultaneously. Batra-Ullah also assume concavity of the production function and decreasing absolute risk aversion to show that the marginal impact of uncertainty in terms of mean-preserving spread is to reduce input demands. Hartman (1975) criticizes the partial approach adopted by Batra-Ullah and in his 1976 article, Hartman also relaxes the assumption that all inputs are chosen before the product price is observed. He claims that the results of Batra-Ullah and Sandmo are rather sensitive to that particular assumption. Korkie(1975) comments that Leland's conclusion is the result of the assumption called the principle of increasing uncertainty. Blair (1974) also discusses some implications of random input prices on the theory of the firm.

Arzac (1976) allows for substitution between expected profit and safety, and proposes the following objective:

Maximize $x + g(\alpha)$, $g' < 0$, $g'' < 0$., where α is the probability that profit falls below a disaster level.

This criterion satisfies the continuity axiom but only its linear form satisfies the independence axiom and is compatible with utility theory (Markowitz (1959) and Arzac (1976)). Arzac also applies the safety-first approaches to the theory of the firm under uncertainty and concludes the following:

a) The overall impact of uncertainty is to lower output.
b) Maximizing the certainty equivalent profit has the same comparative statics properties as the certainty model.
c) If suitable empirical evidence on the firm's past responses to changes in the profit tax rate and in lump sum taxes and subsidies are available, then an almost complete discrimination among the alternative criteria can be made.

A survey of stochastic dominance principle which is used in comparative statics analyses of this paper is found in Levy (1992). Examples of various applications of this principle in investment decision making are available in Levy and Robinson (1998), Kim (1998) and Kjetsaa and Kieff (2003). Empirical works on this principle are reported in Porter and Gaumnitz(1972) and Barret and Donald (2003). Gotoh and Konno (2000) study relationship between Third Degree Stochastic Dominance and Mean –Risk Analyses.

PURPOSE OF THE PAPER AND BASIC ASSUMPTIONS OF THE MODELS

In this paper we study the choice of inputs of a competitive firm facing uncertainty in the product price, assuming that the firm has a subjective probability distribution for the product price.

First, we list the basic assumptions underlying all the models to follow. Second, we study the general risk formulation and derive some comparative static results. Third, the mean-standard deviation model is used which gives stronger results. Finally, we compare the results of the mean-risk models with those of the corresponding expected utility models.

Basic Assumptions

(a) We consider two variable inputs, capital (K) and labor (L).
(b) There is no fixed cost.
(c) The firm is a price taker in the probabilistic sense for its product price (p). The subjective probability distribution of p is F (p), the probability density function is f(p), and the mean is $\overline{p}$.
(d) The prices of inputs (w for L, and r for K) are given parameters with certainty to the firm.
The following assumptions are about the production function x = g(L,K), where x is the output.
(e) $g_K = \frac{\delta x}{\delta K} > 0$, and $g_L = \frac{\delta x}{\delta L} > 0$.
(f) $g_{KK} < 0$ and $g_{LL} < 0$.
(g) $g_{LL} g_{KK} - g^2_{LK} > 0$.
Assumptions (f) and (g) imply that the production function is strictly concave. Concavity of the production function also implies convexity of the isoquants.
(h) We assume $g_{LK} \geq 0$; that is, weak complementarity between K and L.

MEAN-GENERAL RISK MODEL

The Model

Profit π is defined as

$$\pi = pg(L,K) - wL - rK. \tag{1}$$

The risk function (ψ), after changing variables from π to p, is

$$\text{risk} = \int_0^{\hat{p}} \psi[t - (px - wL - rK)] f(p) dp, \tag{2}$$

where $\hat{p} = \frac{t + wL + rK}{x}$, that is price equal to the average of target profit (t) plus input costs. As firms face product-price uncertainty, they set different profit targets. So, average profit target is taken into account. The ψ in equation(2) is a function of [t-(px-wL-rK)] or (t - π) and the assumptions about it are:

ψ(0) = 0, ψ'(0) ≥ 0, ψ'( ) > 0 for π < t, and ψ'' ≥ 0.

The decision variables in this model are L and K, which together with the production function also determine the output level. The objective function is given by

$$V = \bar{p}x - wL - rK - \lambda\int_0^{\hat{p}} \psi[t - (px - wL - rK) or (t - \pi)] f(p)dp, \quad (3)$$

where λ is the relative weight of risk component (a measure of risk-aversion). λ indicates the trade -off between mean and risk, that is, the rate of substitution between return and risk. The higher the λ (i.e., the steeper the indifference curve), the more risk averse the individual is. For risk averters $\lambda > 0$, that is the indifference curves are positively sloped. For a risk neutral agent $\lambda = 0$; that is, the agent maximizes return ignoring the risk. The indifference curves for such an individual would be horizontal. The traditional risk premium and the measure of risk-aversion λ are always positively related. In the context of constrained optimization, λ may be viewed as an allocative efficiency parameter showing how a one unit cost reduction results in profit magnification.

First Order Conditions for an Interior Maximum

$$V_L = \bar{p}\ g_L - w - \lambda\int_0^{\hat{p}} \psi'(t - \pi).(w - pg_L)f(p)dp = 0. \quad (4)$$

and

$$V_K = \bar{p}\ g_K - r - \lambda\int_0^{\hat{p}} \psi'(t - \pi).(r - p\,g_K)f(p)dp = 0. \quad (5)$$

Solving equations (4) and (5), we obtain L and K as functions of $\bar{p}$, w, r, t, and λ. Multiplying V_L by g_K and V_K by g_L, and equating the resulting expressions we have

$$\frac{w}{r} = \frac{g_L}{g_K} \text{ or } wg_k = rg_L \quad (6)$$

This equates wage-rental ratio to the ratio of marginal labor productivity to marginal capital productivity. Equation (6) is the well known tangency condition in deterministic models and shows cost minimization. This result will be frequently used in simplifying the second order conditions, and in many comparative statics analyses.

Now, we prove that the first order conditions imply the following inequalities:

$$\bar{p}\ g_L > w, \quad (7)$$

$$\bar{p}\ g_K > r, \quad (7')$$

$$\int_0^{\hat{p}} \psi'(w - pg_L)f(p)dp > 0, \quad (8)$$

and

$$\int_0^{\hat{p}} \psi'(r - pg_K)f(p)dp > 0. \tag{8'}$$

From equation (4) we have

$$\bar{p}\ g_L - w = \lambda\int_0^{\hat{p}} \psi'(t-\pi).(w - p\,g_L)f(p)\ dp. \tag{9}$$

where, π (total profit) = px -wL - r K

Case (A): $\dfrac{w}{g_L} \geq \hat{p}.$ In this case the integral in equation (9) is clearly positive since ψ' ≥ $\hat{p}.$ Therefore, inequality (7) immediately follows.

Case (B): $\dfrac{w}{g_L} < \hat{p}.$ In this case the integral in equation (9) can be written as

$$I = \int_0^{\frac{w}{gL}} \psi'(t-\pi).(w - pg_L)f(p)\ dp + \int_{\frac{w}{g_L}}^{\hat{p}} \psi'(t-\pi).(w - pg_L)f(p)dp \tag{10}$$

Applying the mean value theorem into equation (10) we obtain for $\hat{\pi}$ and $\hat{\hat{\pi}}$ such that, π(0)

$< \hat{\pi} < \pi(w/\ g_L)$ and $\pi(\dfrac{w}{g_L}) < \hat{\hat{\pi}} < \pi(\hat{p}),$

$$I = \psi'(t-\hat{\pi})\int_0^{\frac{w}{gL}} (w - pg_L)f(p)dp + \psi'(t-\hat{\hat{\pi}})\int_{\frac{w}{g_L}}^{\hat{p}} (w - pg_L)f(p)dp. \tag{11}$$

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Considering the signs of the integral in equation (11) and the assumption ψ'' > 0, we have

$$I \geq \psi'[t-\pi(w/g_L)]\ \int_0^{\hat{p}} (w - pg_L)f(p)dp. \tag{12}$$

From the definition of an expected value we have

$$\bar{p}\ g_L - w = \int_0^{\hat{p}} (pg_L - w)f(p)dp + \lambda\int_{\hat{p}}^{\infty} (pg_L - w)f(p)dp, \tag{13}$$

$+$

or

$$\int_0^{\hat{p}} (pg_L - w)f(p)dp = (\bar{p}\ g_L - w) - \lambda\int_{\hat{p}}^{\infty} (pg_L - w)f(p)dp. \tag{13'}$$

$+$

If $\bar{p}\ g_L - w \leq 0$ is assumed, then from equation (13') we have

$$\int_0^{\hat{p}} (\, p\, g_L \,- w) f(p) dp < 0, \tag{14}$$

or

$$\int_0^{\hat{p}} (w - p\, g_L) f(p) dp > 0. \tag{14'}$$

However, if $\bar{p}\ g_L - w \le 0$, then from (9) and (12) we have

$$\int_0^{\hat{p}} (w - p\, g_L) f(p) dp \le 0, \tag{15}$$

which contradicts (14'). Therefore, we must have $\bar{p}\ g_L - w > 0$. Thus, we have proved that the first order conditions imply inequality (7).

Now using (6) into (7) to replace w by $(r\, g_L)/\, g_K$ we immediately obtain (7'). It is also clear that inequality (6) and equation (9) directly imply (8). Finally, using (6) into (7) we obtain (8')

Inequalities (7) and (7') show that the expected value of the marginal products of each input is greater than the corresponding input price. We prove below that the inequalities (7) and (7') along with our assumptions $g_{LL} < 0$, $g_{KK} < 0$, and $g_{LK} \ge 0$ imply that each input is employed less in uncertainty than in the corresponding certainty case with the product price equal to $\bar{p}$. This is the overall impact of uncertainty on input demands.

Substituting equation (6) into the first order conditions we obtain

$$V_L = \bar{p}\ g_L - w - \frac{w}{r} B = 0, \tag{16}$$

and

$$V_K = \bar{p}\ g_K - r - B = 0, \tag{16'}$$

where

$$B = \lambda \int_0^{\hat{p}} \psi'(r - p\, g_K) f(p) dp > 0.$$

The first order conditions in the corresponding certainty case are

$$V_L = \bar{p}\ g_L - w = 0, \tag{17}$$

and

$$V_K = \bar{p}\ g_K - r = 0. \tag{18}$$

Thus, the first order conditions of the uncertainty case reduce to those of the corresponding certainty case when B equals zero. Hence, if dL/dB < 0, and dK/dB < 0, then the overall impact of uncertainty is to reduce both input demands.

We have from (17) and (18)

$$\begin{bmatrix} g_{LL} & g_{LK} \\ g_{LK} & g_{KK} \end{bmatrix} \begin{bmatrix} dL/dB \\ dK/dB \end{bmatrix} = \begin{bmatrix} w/r\bar{p} \\ 1/\bar{p} \end{bmatrix} \tag{19}$$

Let the determinant $g_{LL}\, g_{KK} - g_{LK}^2$ be denoted by D, which we have assumed to be positive. Moreover, we have

$$dL/dB = \frac{-1}{\bar{p} g_K D}[-g_L g_{KK} + g_K g_{LK}] < 0, \tag{20}$$

and

$$dK/dB = \frac{-1}{\bar{p} g_K D}[-g_K g_{LL} + g_L g_{LK}] < 0, \tag{20'}$$

since $g_{LL} < 0$, $g_{KK} < 0$, and $g_{LK} \geq 0$. This shows that the overall impact of uncertainty is to reduce input demands. Thus, from the usual assumptions about the properties of the production function it immediately follows that output is less under uncertainty than the corresponding certainty level.

Second Order Conditions

Omitting the limits of integration for convenience, the second order conditions are:

$$v_{LL} = \bar{p}\, g_{LL} - \lambda \int \psi''(w - pg_L)^2 f(p)dp + \lambda\, g_{LL} \int \psi' pf(p)dp$$
$$- \frac{\lambda}{x}\psi'(0)(w - pg_L)^2 f(\hat{p}) < 0, \tag{21}$$

$$v_{KK} = \bar{p}\, g_{KK} - \lambda \int \psi''(r - pg_K)^2 f(p)dp + \lambda\, g_{KK} \int \psi' pf(p)dp$$
$$- \frac{\lambda}{x}\psi'(0)(r - pg_K)^2 f(\hat{p}) < 0, \tag{21'}$$

and

$$v_{LL} v_{KK} - v_{LK}^2 > 0, \tag{21''}$$

where v_{LK} is given by

$$v_{LK} = \bar{p}\, g_{LK} - \frac{\lambda}{g_K} \int \psi''(r - p\,g_K)^2 f(p)d + \lambda\, g_{LK} \int \psi' pf(p)dp$$

$$- \frac{\lambda}{xg_K}\psi'(0)(r - pg_K)^2 f(\hat{p}). \tag{22}$$

The full expression for $v_{LL}v_{KK} - v^2_{LK}$ is

$$v_{LL}v_{KK} - v^2_{LK} = [\overline{p} + \lambda\int \psi' pf(p)dp]^2 D + [-g_L^2 g_{KK} - g_K^2 g_{LL}$$
$$+2g_L g_K g_{LK}]\frac{[p + \lambda\psi' pf(p)dp]}{g_K^2}.\ \lambda\int \psi''(r - pg_K)^2 f(p)dp]$$
$$+ \frac{\lambda}{x}\psi'(0)(r - \hat{p}g_K)^2]f(\hat{p}), \tag{22'}$$

where D and $\hat{p}$ are defined above.

It is clear that the second order conditions are also satisfied by the assumption of a concave production function, which is, however, not necessary for that purpose.

Comparative Static Results

Change in the Expected Price

Let $p^* = p + k$ be a transformation of p. Then the objective function is

$$v = (k + \overline{p})x - wL - rK - \lambda\int_0^{\hat{p}^*} \psi(t - px - kx + wL + rk) f(p)dp, \tag{23}$$

where $\hat{p}^* = \frac{t + w_L + r_K}{x} - k = \hat{p} - k.$ From the first order conditions we obtain

$$\begin{bmatrix} v_{LL} & v_{KL} \\ v_{LK} & v_{KK} \end{bmatrix}\begin{bmatrix} dL/dK \\ dK/dk \end{bmatrix} = \begin{bmatrix} -v_{Lk} \\ -v_{Kk} \end{bmatrix}. \tag{24}$$

We can express $-v_{LK}$ and $-v_{KK}$ as

$$-v_{LK} = -g_L A, \tag{25}$$

and

$$-v_{KK} = -g_K A, \tag{26}$$

where $A = 1 + \lambda\int_0^{\hat{p}^*} \psi' f(p)dp + \frac{\lambda x}{g_K}\int_0^{\hat{p}^*} \psi''(r - p^* g_K)f(p)dp$

$$+\ \frac{\lambda}{g_K}\psi'(0)(r-\hat{p}\,g_K)f(\hat{p}). \qquad (27)$$

Note that $\hat{p}$ appears in the last term instead of $\hat{p}*$ because p + k = $\hat{p}$, when p = $\hat{p}*$.

Now solving for dL/dk and dK/dk from expression (24) we obtain

$$dL/dk = \frac{(-g_L g_{KK} + g_K g_{LK})MA}{D}, \qquad (28)$$

and

$$dK/dk = \frac{(-g_K g_{LL} + g_L g_{LK})MA}{D}, \qquad (28')$$

where

$$M = \bar{p} + k + \lambda\int_0^{\hat{p}^*}\psi' pf(p)dp > 0.$$

Since we have assumed $g_{LK} \geq 0$, $g_{LL} < 0$, and $g_{KK} < 0$, we have

$$-g_L g_{KK} + g_K g_{LK} > 0, \text{ and } -g_K g_{LL} + g_L g_{LK} > 0. \qquad (29)$$

Therefore, the signs of dL/dk and dK/dk will be the same as the sign of A. It is clear from equation (27) that

$$A > 0 \text{ if } w/g_L = r/g_K \geq \hat{p} = (t + wL + rK)/x \qquad (30)$$

Thus, a sufficient condition for both input demands to be positively sloped with respect to the expected price is that

$$\frac{w}{g_L} = \frac{r}{g_K} \geq (t + wL + rK)/x. \qquad (31)$$

Note that the inequality in (31) reduces to

$$\text{Marginal cost (MC)} \geq \text{average cost (AC)}, \qquad (31')$$

when t equals zero.

For the case of nondecreasing AC, inequality (31') necessarily holds. Since both input demands move together, and $g_L > 0$ and $g_K > 0$, the inequality in (31) is also a sufficient condition for the output to rise with a rise in the expected price of the product.

Change in Risk Aversion

From the first order conditions we have

$$\begin{bmatrix} v_{LL} & v_{KK} \\ v_{KL} & v_{KK} \end{bmatrix} \begin{bmatrix} dL/d\lambda \\ dK/d\lambda \end{bmatrix} = \begin{bmatrix} \int_0^{\hat{p}} \psi'(w - pg_L)\, f(p)dp \\ \int_0^{\hat{p}} \psi'(r - pg_K)\, f(p)dp \end{bmatrix} \tag{32}$$

which gives

$$dL/d\lambda = -N[-g_L g_{KK} + g_K g_{LK}]Q/g_K D, and \tag{33}$$

$$dK/d\lambda = -N[-g_K g_{LL} + g_L g_{LK}]Q/g_K D, \tag{33'}$$

where N = $\bar{p} + \lambda \int \psi' pf(p)dp > 0,$ and Q = $\int_0^{\hat{p}} \psi'(r - pg_K)f(p)dp.$ (33")

Since the signs of the expressions inside the brackets in (33) and (33') are positive (according to our assumptions $g_{KK} < 0$, $g_{LL} < 0$, and $g_{LK} \geq 0$), the sign of D is assumed to be positive, N is positive, and Q is positive by inequality (8'), it is clear that the signs of dL/dλ and dK/dλ are negative. As a result dx/dλ is negative.

Change in Own Input Price

The derivatives dL/dw and dK/dr are given by

$$dL/dw = (v_{KK}/D)H - \lambda LN/g_K D[-g_L g_{KK} + g_K g_{LK}]J \tag{34}$$

and

$$dK/dr = (v_{LL}/D)H - \lambda KN/g_K D[-g_K g_{LL} + g_L g_{LK}]J \tag{34'}$$

where v_{LL} and v_{KK} are assumed to be negative, D and N are defined above and are positive, and H and J are given by

$$H = 1 + \lambda \int_0^{\hat{p}} \psi' f(p)dp > 0, \tag{35}$$

and

$$J = \int_0^{\hat{p}} \psi''(\ )(r - pg_K)f(p)dp + ((\psi'(0)/x)f(\hat{p})(r - \hat{p}g_K) > 0 \tag{35'}$$

if $w/g_L = r/g_K \geq (t + wL + rK)/x.$ Therefore, the inequality in (31) is again a sufficient condition for both input demands to be negatively sloped with respect to own price.

Change in the Price of Other Input

It is found that the signs of dL/dr and dK/dw are indeterminate even if inequality (31) is assumed. In the next section, we study the mean-standard deviation model that gives significantly stronger results as compared to the general risk model of this section.

MEAN-STANDARD DEVIATION MODEL

The Model

Let σ be the standard deviation of product price. Then, the standard deviation of profit is equal to x σ , where x equals g(k,L). The objective function is

$$V = \bar{p}x - wL - rK - \lambda\sigma x. \tag{36}$$

First Order Conditions for an Interior Maximum

$$v_L = \bar{p}g_L - w - \lambda\sigma\, g_L = 0, \tag{37}$$

and

$$v_K = \bar{p}\, g_K - r - \lambda\sigma\, g_K = 0 \tag{38}$$

Clearly, the first order conditions imply the following results that we use in comparative static anaqlysis:

$$w/g_L = r/g_K\,, \tag{37'}$$

$$\bar{p}\, g_L = w + \lambda\sigma\, g_L > w, \tag{37''}$$

$$\bar{p}\, g_K = r + \lambda\sigma\, g_K > r, \text{ and} \tag{38'}$$

$$\bar{p} - \lambda\sigma = w/g_L = r/g_K > 0 \tag{38'}$$

The Second Order Conditions

$$v_{LL} = \bar{p}\, g_{LL} - \lambda\sigma\, g_{LL} = [\,\bar{p} - \lambda\sigma\,]\, g_{LL} < 0, \tag{39}$$

$$v_{KK} = \bar{p}\, g_{KK} - \lambda\sigma\, g_{KK} = [\,\bar{p} - \lambda\sigma\,]\, g_{KK} < 0, \tag{39'}$$

and

$$v_{LL}v_{KK} - v^2_{LK} = D = (\,\bar{p} - \lambda\sigma\,)^2\,(\,g_{LL}\, g_{KK} - g^2_{LK}\,) > 0. \tag{39''}$$

It is evident that the concavity of the production function is the necessary and sufficient condition for the second order conditions to be satisfied. In contrast, in the model of the previous section the concavity of the production function is only a sufficient (but not necessary) condition for the second order conditions to be satisfied.

COMPARATIVE STATIC RESULTS

Change in the Expected Price

Let p* = p + k be a transformation of p. Then we have

$$dL/dk = (\bar{p} - \lambda\sigma)(-g_L g_{KK} + g_K g_{LK})/D > 0, \tag{40}$$

$$dK/dk = (\bar{p} - \lambda\sigma)(-g_K g_{LL} + g_L g_{LK})/D > 0, \tag{40'}$$

and

$$dx/dk = (\bar{p} - \lambda\sigma)(-g_K^2 g_{LL} - g_L^2 g_{KK} + 2 g_L g_K g_{LK})/D > 0, \tag{40''}$$

since $\bar{p} - \lambda\sigma$ is positive from equation (38") and $g_{LK} \geq 0$ is assumed. If, however, $g_{LK} < 0$ is allowed, then the necessary and sufficient conditions for the input demands to rise with a rise in the expected price are $(-g_L g_{KK} + g_k g_{LK}) > 0$, and $(-g_K g_{LL} + g_L g_{LK}) > 0$, respectively for L and K. These are the same conditions as would be obtained in the corresponding certainty case for dL/dk > 0 and dK/dk > 0, respectively. Moreover, as in the certainty case, the convexity of the isoquants is both necessary and sufficient for the supply of output to rise with a rise in the expected price.

<u>Change in Risk Aversion</u>

In this model we have from the first order conditions

$$\begin{bmatrix} \bar{p}g_{LL} & \bar{p}g_{LK} \\ \bar{p}g_{KL} & \bar{p}g_{KK} \end{bmatrix} \begin{bmatrix} dL/d\lambda \\ dK/d\lambda \end{bmatrix} = \begin{bmatrix} \sigma g_L \\ \sigma g_K \end{bmatrix}, \tag{41}$$

or

$$dL/d\lambda = -\bar{p}\sigma[-g_L g_{KK} + g_K g_{LK}]/D < 0, \tag{41'}$$

and

$$dK/d\lambda = -\bar{p}\sigma[-g_K g_{LL} + g_L g_{LK}]/D < 0. \tag{41''}$$

Thus, an increase in risk aversion reduces both input demands (and consequently output) in the present model without any further assumptions in addition to our basic assumptions (especially, $g_{LK} \geq 0$).

<u>Change in Own Input Price</u>

We have in this model,

$$dL/dw = g_{KK} \; (\bar{p} - \lambda\sigma)/D < 0, \tag{42}$$

and

$$dK/dr = g_{LL} \; (\bar{p} - \lambda\sigma)/D < 0. \tag{42'}$$

Thus, the effect of an increase in input price is negative on input demand, for both K and L without any further assumption in the present model.

The effects on output supply are shown by the following derivatives:

$$dx/dw = (-\, g_L g_{KK} + g_K g_{LK})\,(\bar{p} - \lambda\sigma)/D < 0, \tag{43}$$

and

$$dx/dr = (-\, g_K g_{LL} + g_L g_{LK})(\bar{p} - \lambda\sigma)/D < 0. \tag{43'}$$

Again, allowing g_{LK} to be negative will give the necessary and sufficient conditions for dx/dw < 0 and dx/dr < 0, as in the case of the corresponding certainty model.

Change in the Price of Other Input

The effects of an increase in one input price on another input demand are shown by the following derivatives:

$$dL/dr = -\, g_{LK} \; (\bar{p} - \lambda\sigma)/D \le 0, \tag{44}$$

and

$$dK/dw = -\, g_{LK}\,(\bar{p} - \lambda\sigma)/D \le 0, \tag{44'}$$

since $g_{LK} \ge 0$ is assumed. Allowing $g_{LK} < 0$ will reverse the sign of the inequalities in (44) and (44').

Change in the Variance Keeping Mean Constant

In this model we have

$$\begin{bmatrix} \bar{p}g_{LL} & \bar{p}g_{LK} \\ \bar{p}g_{KL} & \bar{p}g_{KK} \end{bmatrix} \begin{bmatrix} dL/d\sigma \\ dK/d\sigma \end{bmatrix} = \begin{bmatrix} g_L \\ g_K \end{bmatrix} \tag{45}$$

which gives,

$$dL/d\sigma = -\bar{p}(-g_L g_{KK} + g_K g_{LK})/D < 0, \tag{46}$$

and

$$dK/d\sigma = -\bar{p}(-g_K g_{LL} + g_L g_{LK})/D < 0. \tag{46'}$$

Thus, a mean-preserving spread will reduce both input demands (and consequently output supply) in this model.

CONCLUSIONS

In this section we briefly compare the results of our mean-risk models with those of the expected utility models. In particular we have selected the Batra-Ullah (1974) paper as a typical example of the expected utility models of competitive input demand under product price uncertainty,

(a) As Hartman (1975, 1976) points out, Batra-Ullah incorrectly use the partial approach to conclude that the overall impact of uncertainty is to reduce input demands. In this study we have avoided that mistake in the proof of that result. The proof considers both input demands at the same time and correctly arrives at the result under the basic assumptions of the model (especially the assumption $g_{LK} \geq 0$).

(b) Increased risk aversion is not considered in the Batra-Ullah paper, and it appears that it is quite complicated to analyze the effects of a concave transformation of the utility function with two or more decision variables. In our case, the effect of an increase in risk aversion is easily analyzed and for the general risk model the inequality $w/g_L \geq (t + wL + rK)/x$ is a sufficient condition for negative effects on input demands, whereas for the standard deviation model no further assumption is required.

(c) In addition to the conditions $g_{LL} < 0$ and $g_{KK} < 0$, Batra-Ullah had to assume the strict inequality $g_{LK} > 0$, and decreasing absolute risk aversion in order to obtain a negative slope of the input demand curves with respect to own price. In our mean-standard deviation model no further assumption (except $g_{LK} \geq 0$) is required, whereas in the general risk model the weak inequalities $g_{LK} \geq 0$ and w/ g_L $\geq$ (t + wL + rK)/x are the sufficient conditions.

(d) Similarly, in all other comparative statics analyses it is found that we have $w/g_L \geq (t + wL + rK)/x$ as a sufficient condition whenever Batra-Ullah have decreasing absolute risk aversion as a sufficient condition in their model.

(e) Among others, increases in firm's own input-prices and risk aversion have contractionary effects on input demands and hence on output.

In closing, increasing product-price uncertainty is the result of intensifying market-competition springing from the ongoing waves of globalization, market integration, deregulations weak property rights, etc. To cope with this growingly complex and uncertain business environment, firms have to be cost-effective by efficient allocations and utilizations of indispensable factors of production. To understand the behavior of business firms under these circumstances, a less restrictive mean-variance model in this case is found more helpful than a more restrictive expected utility model as put forward in Batra and Ullah (1974). However, there are other variants of the expected utility approach that are presumably more elegant than that in the above.

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