USING SEASONAL AND CYCLICAL COMPONENTS IN LEAST SQUARES FORECASTING MODELS

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ABSTRACT

Although many articles have been written concerning the improved accuracy of combined forecasts, sometimes the obvious is overlooked. By combining seasonal indices and cyclical factors with other explanatory variables, forecasting models acquire increased accuracy for out-of-sample predictions.. This paper encourages the use of least squares forecasting models with time series components. It also provides new directions for research in combining forecasts. This approach to forecasting is also compared to other popular forecasting methods. Surprisingly, the use of seasonal indices and cyclical factors in least squares equations does not frequent the literature.

INTRODUCTION

The decomposition method of separating time series data into the four components of trend, cyclical movement, seasonal variation, and irregular fluctuations is well known. Indeed, combining these components in a multiplicative manner is one of the oldest methods of forecasting (Barton, June 1941). However, considerable advantages are obtained by including seasonal indices and cyclical factors in a least squares forecasting equation:

$$\hat{Y}_t = b_0 + b_1X_t + b_2S_j + b_3C_t \qquad (1)$$

where X_t are for trend values, C_t are cyclical factors, and S_j are seasonal indices repeated each year. This approach becomes attractive when compared with other forecasting methods.

Equation (2) describes the dummy variable approach to quarterly seasonal variation:

$$\hat{Y}_t = b_0 + b_1X_t + b_2D_2 + b_3D_3 + b_4D_4 + b_5C_t, \qquad (2)$$

where X_t and C_t are defined in (1) above; $D_j = 1$ if quarter j, $j = 2, 3, 4$, 0 otherwise. Equations (1) and (2) have approximately the same accuracy. The dummy variable method of including seasonal variation is described in most econometric textbooks (Greene, 2000; also Ramanathan, 2002). Although (1) has the advantage of using a single index variable, its applications to forecasting does not frequent the literature. When describing monthly seasonal variation, the dummy variable approach must employ 11 binary variables as compared to the one seasonal index variable in (1). This alone has considerable computational and methodological implications. Time series components in unrestricted least squares models are highly conducive to judgement modification operations thereby increasing the accuracy of out-of-sample forecasts. Hence, this approach extends the capabilities of combining forecasts using unrestricted least squares coefficients as weights (Granger and Ramanathan, 1984). An example is given which compares (1) with other forecast methods.

CONCEPTS AND NOTATIONS

Although there are exceptions, the accuracy obtained by using (1) over the traditional decomposition method conforms to intuition. Least squares estimates by (1) are more accurate than non-least squares estimates from

$$T_t{*}S_j{*}C_t \tag{3}$$

where S_j and C_t are defined in (1) above, and T_t are trend estimates;

$$T_t = b_0 + b_1X_t \tag{4}$$

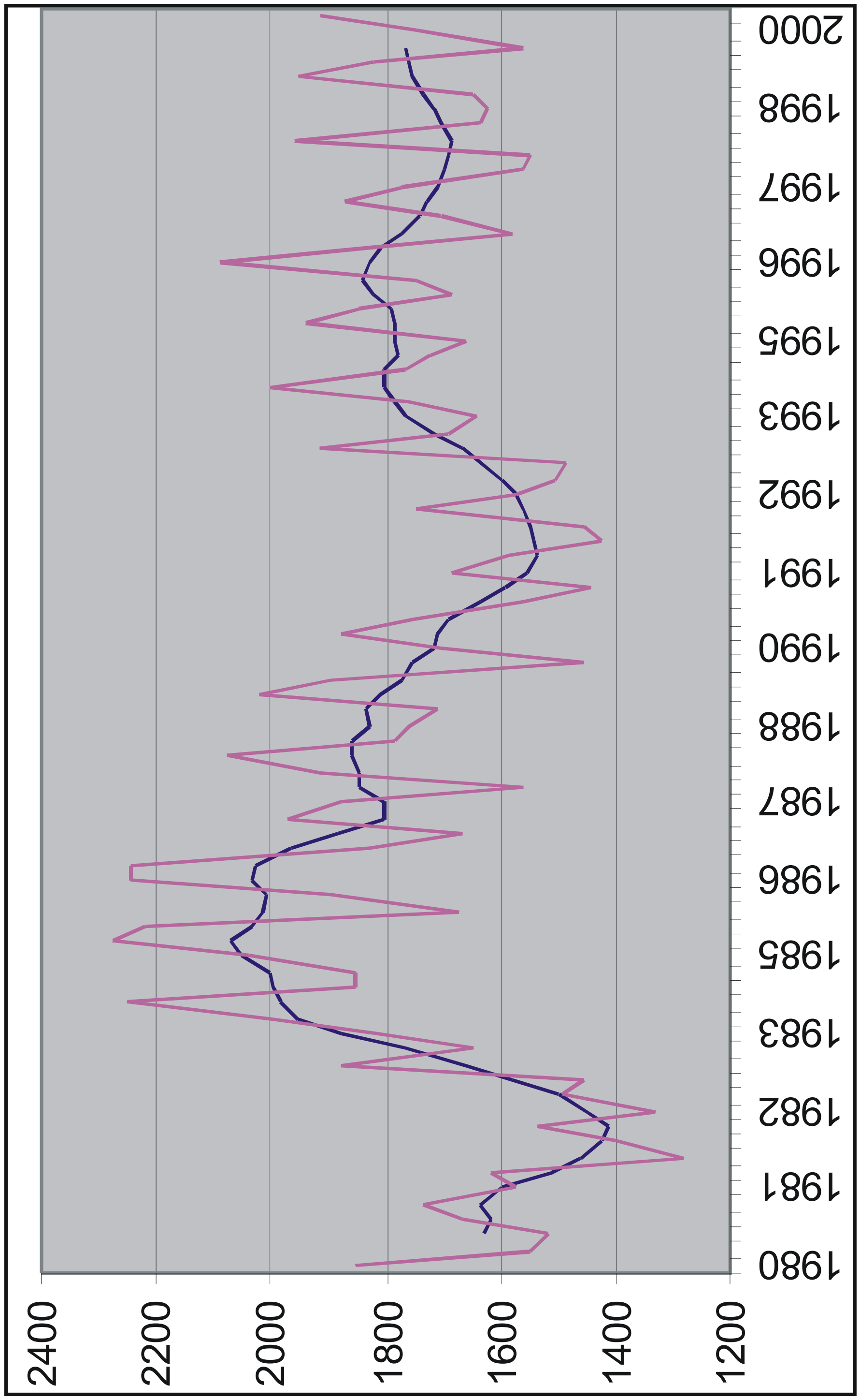
Moving Seasonals. When using a constant seasonal index, it is assumed the seasonal variation is not moving – is not becoming stronger or weaker. However, if this assumption is incorrect or if there is considerable dispersion in the seasonal factors for a particular period, a moving seasonal index needs to be constructed. A moving seasonal index is described in older textbooks (Croxton and Cowden, 1955) and has been used for decades by the U.S. Bureau of the Census. Moving seasonal indices are used when the average seasonal indices do not adequately describe current seasonal variations. When forecasting future Y values, moving seasonal indices may be obtained subjectively. This allows the model to posses judgement modification capabilities.

Cyclical Movement. Cyclical factors are computed from the ratio $C_t = Y_{Dt}/\hat{Y}_{Dt}$, where Y_{Dt} and $\hat{Y}_{Dt}$ are the actual and trend values of deseasonalized data. These factors may also be obtained subjectively when forecasting future values of Y.

COMPARING FORECASTING METHODS

In order to fix concepts, the following example is given. This example compares the accuracy of using seasonal and cyclical components in least squares models with other forecasting methods.

Figure 1. Domestic Car Sales With Centered Moving Averages



Domestic Car Sales Example.

Wilson and Keating (2002) in *Business Forecasting* use domestic car sales (DCS) from 1980 to 1999 in comparing the accuracy of various forecasting methods. Figure 1 reveals DCS have little trend but a substantial cyclical movement. The root mean square error (RMSE) for each method is shown in Table 1; where RMSE is the square root of the sum of squares error divided by n, RMSE = $(SSE / n)^{1/2}$. A measure of goodness of fit is obtained from the historical RMSE. This includes quarterly data from 1980 through 1998. A measure of goodness of prediction is obtained from the holdout RMSE. This includes quarterly data for 1999. These quarters are not used in deriving the various models and are considered out-of-sample data.

Observe from Table 1, the least squares models with constant and moving seasonals (methods 9 and 10) are top contenders with regard to the goodness-of-fit (historical RMSE) measures. The least squares dummy variable model and the combined forecast model (methods 8 and 11) also performs well. These models perform even better with regard to the goodness-of-prediction (holdout RMSE) measures.

TABLE 1
COMPARING FORECASTING METHODS: RMSE FOR DOMESTIC CAR QUARTERLY SALES

Forecasting Method	**Historical RMSE**	**Holdout RMSE**
1. Winter's Exponential Smoothing	144.45	61.79
2. Holt's With Seasonal	200.46	62.21
3. Deseasonalized DCS with DPI 1990-1998*	96.98	87.93
Deseasonalized DCS with DPI 1980-1998	190.62	61.25
4. Multiple Regression	107.89	162.59
5. Time-series Decomposition	195.50	181.20
6. ARIMA(1,1,0) (1,2,1)	172.56	120.09
7. Combined Winter's and Regression	141.69	53.79
8. Dummy Variable Method	91.68	48.65
9. Least Squares Constant Seasonals	91.76	48.92
10. Least Squares Moving Seasonals	30.26	10.32
11. Combined Methods 5 and 10	25.90	11.07

*Method 3 used 1990 through 1998 data and is not a true comparison

Least Squares Models With A Moving Seasonal Index.

The least squares constant seasonal model (method 9 in Table 1) is described by (1) above. The least squares moving seasonal model (method 10 in Table 1) is identical to (1) except moving seasonals S_t are used for the seasonal indices;

$$\hat{Y}_t = b_0 + b_1X_t + b_2S_t + b_3C_t. \tag{5}$$

Observe, Y_t are the estimated DCS values, X_t are for trend values, C_t are the cyclical factors, and S_t are the quarterly seasonal factors adjusted to equal 4.0 for each year.

Figure 2a reveals the seasonal factors for March are not adequately represented by their average. Therefore, a moving seasonal factor for March is projected for the year 1999. This projection may be subjective by deciding whether the future March seasonal factor will be more, less, or equal to the previous March seasonal factor. Other quarterly seasonal factors are created subjectively in a similar manner (Figure 2). By observing the cyclical movement in Figure 1, one decides whether future C_t values will increase, decrease, or remain the same. A knowledge of the subject matter is imperative in subjectively selecting a reasonable value for C_t when forecasting future out-of-sample Y values. Hence, the model possesses judgement modification capabilities in selecting future values for S_t and C_t.

Figure 2a. March Seasonals

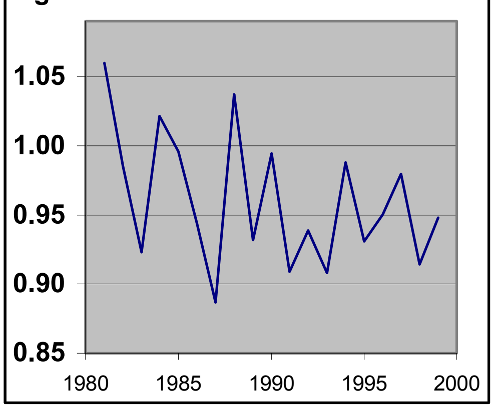


Figure 2b. June Seasonals

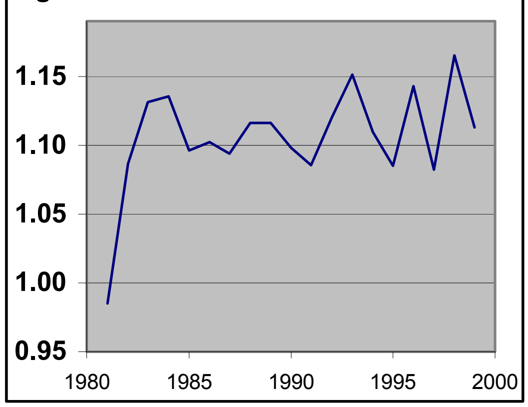


Figure 2c. September Seasonals

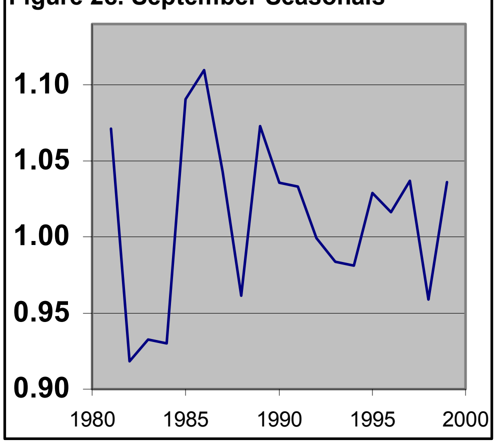
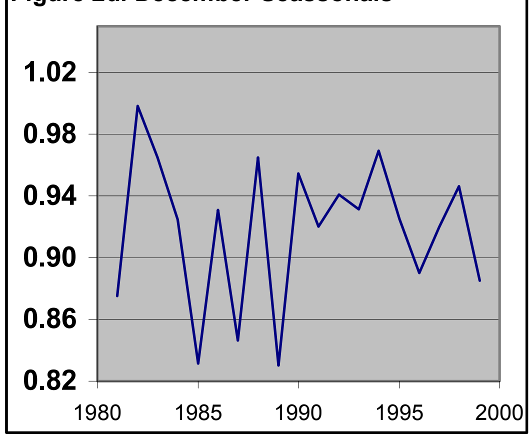


Figure 2d. December Seassonals



Other Methods.
In Table 1, method 3 used 1990 through 1998 data when regressing DCS with DPI;

$$DCS = b_0 + b_1 DPI.$$

A true comparison requires the data to be from 1980 through 1998. Method 4 regresses DCS with the following:
DPI: disposable personal income, DPI^2: DPI squared PR: prime interest rate
$D_j = 1$ if quarter j, j = 2,3,4, 0 otherwise, X: trend, X^2: trend squared,
Index: University of Michigan Index of Consumer Sentiment

$$\hat{Y}_i = b_0 + b_1 DPI + b_2 DPI^2 + b_3 PR + b_4 D_2 + b_5 D_3 + b_6 D_4 + b_7 Index + b_8 X + b_9 X^2 \quad (6)$$

Method 8 is the dummy variable method described by (2). The RMSE for method 8 (also method 4) is downward biased. The sum of squares error (SSE) in RMSE = $(SSE/n)^{1/2}$ will not increase and usually decreases when an additional variable enters the model. Hence, the more variables use in the model, the lower SSE becomes thereby making RMSE lower.

Combined Forecasts. By including the multiplicative product described by (3) in least squares equations (1) or (5), increased accuracy is usually obtained (see method 11 in Table 1):

$$\hat{Y}_t = b_0 + b_1 X_t + b_2 S_t + b_3 C_t. + b_3 W_t, \quad (7)$$

where $W_t = T_t*S_j*C_t$ is defined in (3). Most authors (Bates and Granger, 1969; Batchelor and Dua, 1995; also Bopp, 1985; Granger, 1989) agree that combined forecasts outperform forecasts from a single method. Hence, a major advantage of (1) and (5) is that additional forecasts can easily be included in the least squares equation.

DISCUSSION

The inclusion of time series components in least squares models offers a new approach to forecasting. Indeed, using seasonal indices and cyclical factors as explanatory variables in least squares models provide an excellent method of combining forecasts. The accuracy of these models is usually increased when a moving seasonal index, rather than a constant seasonal index, is employed.

Unrestricted Least Squares. Granger and Ramanathan (1984) argue that combining forecasts from several models will outperform forecasts from a single model. When biased forecasts are included in a least squares equation, the intercept adjusts for the bias. Hence, it is important to use least squares equations with an intercept. The authors totally agree with Granger and Ramanathan that the common practice of obtaining a weighted average of alternative forecasts should be abandoned in favor of least squares equations with an intercept. However, the authors carry the research one step further by using time series components that are conducive to judgement modifications.

Judgement Modification is including the forecaster's knowledge of the subject matter into the model. Indeed, Armstrong 1978, Mahmoud 1984, and Young 1982, to name a few, state that judgement modification is a vital and necessary ingredient of forecasting. It has been shown that judgement modification should modify the components of a forecast and not the computed forecast value itself (Lawrence, et al. 1986). Edmundson (1990) uses a modified trend in obtaining the computed forecast value. The modification is structured by using graphs to help subjectively select the slope of the trend. Since past influences do not necessary continue in the future, judgement modification is an excellent method of increasing the accuracy of out-of-sample predictions. Although moving seasonal indices have been used for decades, subjective estimates of moving seasonal indices in least square forecasting models are unique. This judgement modification procedure is an excellent way of increasing the accuracy of out-of-sample forecasts. The same rationale can be made for using subjective cyclical indices.

Model Specification. By using unrestricted least square, combined forecasts models always produce the most accurate in-sample fitted values. (Granger and Ramanathan 1984). However, this superior accuracy does not always continue with out-of-sample predictions. This is because influences of the past do not necessary continue in the future. It is our contention that these suboptimum out-of-sample performances are caused by specification errors rather than estimation errors. Therefore, attention is directed to how the inclusion of time series components in unrestricted least squares equations enhances combined forecasts. This enhancement comes from the inclusion of time series components that are highly conducive to structured human judgmental modifications.

CONCLUSION

This paper provides new directions for research in combining forecasts. Although the use of unrestricted least squares in combining forecasts yields superior accuracy for in-sample fitted values, it sometimes yields suboptimum accuracy for out-of-sample predictions. This paper advocates treating suboptimum performances as specification errors rather than estimation errors. In doing so, the use of time series components is highly recommended. Since time series components are highly conducive to judgement modifications, their inclusion in combined forecast models should be seriously considered. Indeed, their inclusion helps insure increased accuracy for out-of-sample predictions.

Be cognizant that a single monthly seasonal index variable accomplishes what takes 11 dummy variables to achieve. This alone has far reaching methodological implications. Therefore, the inclusion of time series components allows least squares forecasting models to acquire increased accuracy, broad applicability, and judgement modification capabilities. These attractive features will make the use of time series components in least squares equations increasing popular among forecasters.

REFERENCES

Armstrong, J.S. (1978) *Long-Range Forecasting: From Crystal Ball to Computer,* New York: John Wiley.

Barton, H.C. Adjustment for Seasonal Variation, *Federal Reserve Bulletin*, June 1941.

Batchelor, R., P. Dua. 1995. "Forecast Diversity and the Benefits of combining Forecasts," *Management Sciences.* 41 68-75.

Bates, J. and C. Granger (1969), "The Combination of Forecasts," *Operations Research Quarterly*, 20

Croxton, Frederick E. and D. J. Cowden (1955) *Applied General Statistics*, Englewood Cliffs, N.J., Prentice-Hall, Inc.

Edmundson, R. 1990. "Decomposition. "A Strategy for Judgemental Forecasts," *J. Forecasting*, 9, 4, 305

Granger, C. (1989), "Combining Forecasts--Twenty Years Later", *Journal of Forecasting.*

Granger, C. and Ramanathan (1984), "Improved Methods of Combining Forecasts", *Journal of Forecasting* 3, 197 - 204.

Greene, W. (2000), *Econometric Analysis*, 4th ed, Upper Saddle River, NJ, Prentice-Hall, Inc.

Kennedy, Peter (2003), *A Guide To Econometrics*, 5 th ed, Cambridge, Massachusetts, The MIT Press

Lawrence, M. J., Edmundson, R. H. and O'Connor M. J. 1986. "The Accuracy of Combining Judgmental and Statistical Forecasts," *Management Science*, 32, 1521-1532.

Mahmoud, E. (1984) "Accuracy in Forecasting: A Survey," *Journal of Forecasting* 3, 139-159.

Ramanathan, R. (2002), *Introductory Econometrics With Applications*, 5th ed, Mason, South-Western

Wilson, J.and B.Keating (2002), *Business Forecasting*, 4th, NY, McGraw-Hill.

Young, R. (1982),Forecasting With An Econometric Model: The Issue of Judgmental Adjustment, *Journal of Forecasting* 1, 189-204.