A SURVEY OF AGGREGATE MEASUREMENT OF POVERTY

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ABSTRACT

Recent reviews of literature on the measurement of poverty were done before 1994 so they did not witness the new approach for the poverty measure introduced that year. In this paper, the author surveyed the traditional poverty measures, Sen's poverty measure and similar subsequent literature along with the new approach for the poverty measure which is the neighborhood adjusted poverty measure.

Introduction:

Poverty is of great concern to policy makers, researchers, and almost everyone in the United States. With poverty rates exceeding 20% in some states[1] and exceeding 40% in some poverty areas within the states[2], researchers began to look closely at poverty and the way we measure it.

Sen has made a major contribution in setting up a reasonable poverty measure. In 1976, he criticized the traditional poverty measures existing at that time because they were not sensitive to the income distribution of the poor (i. e., they were not sensitive to the intensity of poverty among the poor). He then introduced three axioms for any poverty measure to satisfy, and proposed and justified a poverty measure that satisfies these axioms. Since then, similar measures have been introduced in the literature.

In 1994, Abdel-Raouf introduced a new approach for the poverty measure that takes into account the nonmonetary aspects of poverty so that the poverty measure will be sensitive to the neighborhood characteristics in which the person lives.

This paper presents a survey of aggregate poverty measures introduced in the literature. Aggregate poverty measures estimate the poverty level in a group of social units (individuals, families or households). Literature on poverty measures is divided into three groups:

- The Traditional Measures.
- Sen's Measure and Similar Subsequent Measures.
- The Neighborhood Adjusted Poverty Measure.

Section two of this paper discusses the traditional poverty measures and their shortcomings. Section three presents Sen's measure and similar subsequent poverty measures. Section four deals with the neighborhood adjusted poverty measure and explains why it is important to include the neighborhood characteristics in measuring poverty. Section five concludes the research.

TRADITIONAL POVERTY MEASURES:

Before 1976, there were basically two groups of poverty measures: the first one was based on counting the poor and the second one was based on the income short-fall from the poverty line for all poor people which is called the aggregate poverty gap.

The following notations will be used throughout this paper:

- S is the set of people in the community with n individuals, $i = 1, 2, \ldots, n$.

- Each person receives income y_i, y represents the income distribution for the whole community, $y=(y_1, y_2, \ldots, y_n)$.

- The set of the poor is $T(y,\pi) \subset S = \{i \in n \mid y_i \leq \pi\}$

- The number of poor is $q(y,\pi)$, i.e., q is the number of people in set T.

- The poverty measure $P(y, \pi)$ is a function that specifies a value of $P(y, \pi)$ to each income distribution y given a specific poverty line π.

The First Group:

This group includes two measures:

1. The aggregate head count $q(y, \pi)$.
2. The head count ratio $H(y, \pi) = [q(y, \pi)]/n$.

These two measures suffer from three disadvantages:

1. They are not sensitive to the income level of the poor. This means that when the income of a poor person decreases, the poverty measure will not change.
2. They are not sensitive to the income distribution of the poor. This means that if a poor person transfers some of his income to a richer poor person who stays poor after the transfer, the poverty measure will not change.
3. They are not sensitive to the neighborhood in which the family lives which has proven to affect the life chances of its individuals.[3]

The Second Group:

This group includes three measures:

1. The aggregate poverty gap $g(y,\pi)$ which is the summation of the individual poverty gaps, i. e. $g(y,\pi) = \sum_{i \in T} g_i(y, \pi)$ where the i^{th} individual poverty gap $g_i(y, \pi)$ is defined as $g_i(y, \pi) = \pi - y_i$.
2. The average poverty gap g/q.
3. The income gap ratio-as Sen called it- $I(y, \pi)$, where $I(y, \pi) = g/q\pi$.

These three measures (g, g/q, and I) do not suffer from the first previously mentioned disadvantage since when the income of a poor person falls, his poverty gap will increase, as well as the poverty measure. However, they still suffer from the second disadvantage since when a poor person transfers some of his income to a richer poor person who stays poor after the transfer, the transferer's poverty gap will increase by the same amount as the decrease in the recipient's poverty gap, so that the aggregate poverty gap will not change. They also still suffer from the third disadvantage since they are only concerned with the monetary income as a measure of poverty.

SEN'S MEASURE AND SIMILAR SUBSEQUENT LITERATURE:

Sen's Measure:

Sen was concerned with the first and second disadvantages of the traditional poverty measures so in 1976[5] he introduced three axioms that any poverty measure should satisfy. These axioms are:

Focus Axiom: The poverty measure should be concerned with the incomes of the poor only, since the poverty measure is a characteristic of the poor.

Monotonicity Axiom: The poverty measure has to increase whenever the income of a poor person decreases, ceteris paribus.

Weak Transfer Axiom: When a poor person transfers some of his income to a richer poor person who stays poor after the transfer, the poverty measure has to increase.

The first two axioms are clear enough but the third one needs justification. There are two kinds of arguments. *The first one*[6] is that since the marginal utility of income is positive and decreasing, then the decrease in the transferer's utility is greater than the increase in the recipient's utility. The net change will be a decrease in utility which should increase the poverty measure. *The second one* is based on the concept of relative deprivation; since the poorer person is more deprived than the richer one and the transfer goes from the poorer to the richer, then the overall level of relative deprivation will increase which should increase the poverty measure.

Sen found that none of the existing measures was satisfying his axioms, so he introduced a new measure that satisfies them.

Sen's poverty measure is:

$$P_s(y,\pi) = 2/[(q+1)\, n\pi] \sum_{i\in T} g_i r_i(y,\ \pi), \qquad (1)$$

where $r(y,\ \pi)$ is the ranking of the poor such that:

$$r_i > r_j \qquad \text{if} \qquad g_i(y,\pi) > g_j(y,\pi).$$

Person i has a higher ranking than person j if his poverty gap is greater than person *j's* poverty gap, which implies that person i has a lower income than person j.

Sen based his measure on a weighted summation of the individual poverty gaps after normalizing it with the normalization factor $2/(q+1)n\pi$. The weight that Sen used is the ranking of the poor.

Sen's measure satisfies his three proposed axioms, as explained below:

Focus *axiom:* If we have two income distributions x and y such that $x_i = y_i\ \forall i \in T$ and $x_i \neq y_i$ for some $i \in T \Rightarrow g_i(x,\pi) = g_i(y,\ \pi)\ \forall\ i \in T \Rightarrow r_i(x,\ \pi) = r_i(y,\ \pi) \Rightarrow P_s(x,\ \pi) = P_s(y,\ \pi)$.

Monotonicity axiom: If the income of a poor person decreases $\Rightarrow$ his poverty gap g_i increases $\Rightarrow g_i r_i$ is higher $\Rightarrow P$ is higher.

Weak transfer axiom: If a poor person A transfers some of his income to a richer poor person B who stays poor after the transfer $\Rightarrow g_A$ increases by the same amount as g_B decreases but since $r_A > r_B$ because person A is poorer than person $B \Rightarrow g_A r_A$ increases by more than the decrease in $g_B r_B$ so that the net change in $g_i r_i$ is positive $\Rightarrow g_i r_i$ is higher $\Rightarrow P$ is higher.

Sen also showed that his measure can be equivalently written as:

$$P_s(y,\ \pi) = H\{I+G(1-I)\ [q/(q+1)]\}^{9} \qquad (2)$$

where H is the head count ratio
and G is the Gini measure of inequality among the poor.

Poverty Measures Introduced in the Subsequent Literature:

The author will follow Foster (1984) in dividing the subsequent literature on poverty measures into three groups: *The First Group* varies the weight and/or the normalization factor of Sen's measure. *The Second Group* replaces the Gini measure of inequality used by Sen with another inequality measure. *The Third Croup* -which began in 1984- is concerned with poverty in a subgroup of the population and how it contributes to the total poverty.

The First Group:

This group includes, but is not limited to, Anand (1977), Thon (1979) Kakwani (1980a), Kundu and Smith (1983), Donaldson and Weymark (1986), Atkinson (1987), and Shorrocks (1995).

Anand (1977)

Anand used different weight and normalization factor to obtain two different formulas for the poverty measure.
Instead of using the income of the poor, Anand used "equally distributed income"[10] so the weighted income gap equals the difference between the poverty line and the equally distributed equivalent income of the poor.
Anand also used another normalization factor so that if the incomes of the poor are equal, then the poverty measure is equivalent to the ratio of the poverty gap to the total income of the community
{i. e., $(q/n)[(\pi - \bar{y}_p)/(\bar{y})]$},
where : $\bar{y}$ is the mean income of the population.
and $\bar{y}_p$ is the mean income of the poor.

Anand's poverty measure is:

$$P_a = (q/n)(1/\bar{y})[\pi - \bar{y}_p(1 - G)]. \qquad (3)$$

Anand expressed his measure in terms of Sen's measure as:

$$P_a = (\pi / \bar{Y}_n) P_s. \qquad (4)$$

Anand modified the normalization factor a little more to obtain a second measure of poverty, where he expressed the poverty gap as a ratio of total income of the nonpoor. Let the mean income of the nonpoor be $\bar{y}_n$, then Anand's second poverty measure is:

$$P'_a = (\pi / \bar{y}_n) P_s \qquad (5)$$

The motivation for Anand's second measure P'_a was to eliminate poverty through a direct transfer of income from the nonpoor to the poor, which puts a burden on the non-poverty group since it represents the proportionate reduction in their income needed to close the poverty gap if we want to close the poverty gap through redistribution alone.[12]

Note that while Anand's measures satisfied both the monotonicity and weak transfer axioms, they violated the focus axiom because if the income of the nonpoor increases $\Rightarrow$ $\bar{y}$ and $\bar{y}_n$ increase $\Rightarrow$ P_a and P'_a decrease. However, he explained this as follows: when the income of the nonpoor increases, a smaller fraction of the community's income is now needed to eliminate poverty; therefore, the task is easier and the measure decreases.

Thon (1979):

Thon was concerned with the "transfer axiom" introduced by Sen in 1976. The transfer axiom states "a transfer of income from a poor person to anyone richer should increase the poverty measure, ceteris paribus". This axiom does not put any constraint on the number of the poor, so the poverty measure has to increase even if the transfer results in a decrease in the number of poor. In contrast, the weak transfer axiom introduced by Sen in 1977 puts a restriction on the number of poor and requires the poverty measure to increase only if the transfer results in no change in the number of poor.

In 1977, Sen noted that his poverty measure (P_S) does not satisfy the transfer axiom. Thon considered this to be a disadvantage of the measure since it implies that a policy of redistributing income from the poorest poor to the richest poor person who crosses the poverty line after the transfer decreases the poverty measure.

Thon proposed *six requirements* for any poverty measure to satisfy. These requirements were:[13]

1. Given other things, a transfer of income from a poor person to anyone richer should increase the poverty measure.
2. Given other things, a transfer of income to a strictly poor person from a rich person who stays rich after the transfer should decrease the poverty measure.
3. Given other things, an increase in a strictly poor person's income should decrease the poverty measure.

By analyzing these requirements, we see that the first is nothing more than the transfer axiom introduced by Sen in 1976. The second is the weak transfer axiom but with a progressive transfer[14] instead of a regressive[15] one as used by Sen in 1977. The third is the monotonicity axiom introduced by Sen in 1976. However, Thon ignored the focus axiom as a requirement for any poverty measure to satisfy.

Based on his requirements, Thon proposed the following poverty measure, which satisfied all his requirements:

$$P_t(y, \pi) = \frac{2}{(n+1)n\pi} \sum_{i \in T} g_i(r_i + n - q). \qquad (6)$$

Thon arrived at this measure by changing the weighting scheme used by Sen through adding the number of the nonpoor $(n - q)$ to each weight and renormalizing. These weights can be interpreted as rankings of the poor among *all* persons in the community[16] so the poorest person has a rank of n and the richest poor person has a rank of $(n - q + 1)$.[17]

Thon achieved his goal and provided us with a measure that satisfies the transfer axiom. However, the satisfaction of this axiom has been debated. Sen, for example, does not think that the poverty measure should satisfy the transfer axiom. Kundu and Smith (1983) showed that if the poverty measure is concerned with the population size and the number of poor and nonpoor in the population[18], then it cannot satisfy the transfer axiom.

Kakwani (1980a):

Kakwani wanted to provide a generalization of Sen's measure. He criticized Sen's measure for failing to satisfy the transfer-sensitivity axioms that he proposed. Therefore, he made small changes in the weighting scheme and the normalization factor in order to obtain a different formula for the poverty measure and to satisfy the proposed axioms. Kakwani's proposed axioms are:[19]

Monotonicity-Sensitivity Axiom: A reduction in the income of a poor person causes a larger increase in the poverty measure than if the reduction is in the income of a less poor person.

Note the difference between Sen's monotonicity axiom and this axiom; in the former, the poverty measure increases when there is a reduction in the income of a poor person, while in the latter the increase in the poverty measure should be larger if the same amount of income is taken away from a poorer person.

Kakwani noted that the monotonicity-sensitivity axiom is not independent of Sen's monotonicity and transfer axioms. In particular, the monotonicity and the monotonicity-sensitivity axioms imply the transfer axiom.

Transfer-Sensitivity I Axiom: A transfer of income between two poor persons a fixed number of ranks apart should have a greater effect on the poverty measure the higher the ranking of the transferer.

Kakwani noted that Sen's measure violates this axiom since it gives the same weight to an income transfer among poor persons of different ranking.

Transfer-Sensitivity II Axiom: A transfer of income between two poor persons whose incomes differ by a fixed amount should have a greater effect on the poverty measure the lower the income of the transferer.

Kakwani noted that Sen's measure violates this axiom as well.[20]

The difference between transfer-sensitivity axioms I and II is that in the former it is the number of ranks that is fixed between the transferer and the recipient, while in the latter it is the difference in incomes that is fixed between them.

Kakwani's Poverty Measure:

Kakwani changed Sen's weighting scheme by raising each weight to a power $m \geq 0$ then renormalizing to get this measure:

$$P_k(y, \pi) = \frac{q}{n\pi\phi_q(m)} \sum_{i \in T} g_i r_i^m, \qquad (7)$$

where $\phi_q(m) = \sum_{i=1}^{q} i^m$.

Equation (7) shows that when $m=0 \Rightarrow P_k(y,\pi) = HI$, and when $m=1 \Rightarrow P_k(y, \pi) = P_s(y, \pi)$. For $m>0$, $P_k(y, \pi)$ satisfies the three axioms introduced by Sen and that for m sufficiently greater than one, his measure satisfies both transfer-sensitivity axioms.[21]

Clark et al. (1981) claimed that the choice of m depends on the income distribution we are looking at. They also mentioned that this will not be the case if n is fixed.

Foster (1984) mentioned that the appropriate m depends on the size of the population n, and warned us that for any given m, we can find a population size n that makes Kakwani's measure $P_k(y, \pi)$ a violation of transfer-sensitivity II axiom.

Kundu and Smith (1983):

Kundu and Smith criticized the existing poverty measures (beginning with Sen 1976) for looking at income changes within a *fixed* population, which cannot be used to compare poverty levels in two different countries or in the same country at different points in time where the population size is different.[22] Therefore, they introduced the "population monotonicity" axioms and showed that no poverty measure satisfies these axioms and Sen's transfer axiom (or the weaker form of it) simultaneously.

Kundu and Smith looked at a wider range of poverty measures that uses all the population, not just the poor, and defined the poor as those with income strictly less than the poverty line. They developed three axioms for any poverty measure to satisfy. These axioms are:

Upward Transfer Axiom: An order-preserving transfer[24] of income from a poor person to anyone richer should not decrease the poverty measure.

This axiom is the weakened form of Thon's (1979) requirement number one as well as the weakened form of Sen's transfer axiom.

Poverty Growth Axiom: If a poor person is added to the population, ceteris paribus, then the poverty measure should increase.

Nonpoverty Growth Axiom: If a person with income above the poverty line is added to the population, ceteris paribus, then the poverty measure should decrease.

Kundu and Smith found that none of the existing poverty measures satisfies these three axioms.[25] Furthermore, they showed that no poverty measure[26] can satisfy these three axioms and any poverty measure that does satisfy these three axioms has to be monotonically decreasing.[27] While Kundu and Smith proved that for any poverty measure to satisfy the "population monotonicity" axioms, it has to violate the transfer axiom,[28] Sen and Foster see this contradiction differently.[29]

Donaldson and Weymark (1986):

Donaldson and Weymark introduced many axioms and investigated the interrelationships among various combinations of these axioms. They focused on poverty indices that satisfy the focus axiom with a fixed population and a fixed poverty line. Donaldson and Weymark introduced new poverty indices from the already existing ones by changing the weighting scheme associated with them or by adding (or subtracting) a positive constant to the income gaps so that the obtained indices satisfied various combinations of their axioms. However, they did not provide us with a poverty measure that satisfied all of their axioms simultanously.[31]

Some of the axioms they considered were already in the literature and some were not. The focus axiom is essential to them and they considered only poverty measures that satisfy it. Donaldson and Weymark also considered two monotonicity axioms, four transfer axioms and continuity axiom.

The monotonicity axioms they considered are the weak (downward) monotonicity axiom and the strong (upward) monotonicity axiom. *The former* is what has been used by Sen (1976) and *the later* extends it to the case when the number of the poor are not fixed. It requires the poverty measure to decrease if a poor person's income increased, ceteris paribus, and that increase is sufficient to make this person rich.

The first two of the transfer axioms deal with a fixed number of the poor, while the third and fourth ones do not. The former includes the minimal and weak

transfer axioms, while the later includes the strong upward and strong downward transfer axioms. *The minimal transfer axiom* is the same as the weak transfer axiom introduced by Sen. *The weak transfer axiom* extends it to a situation where the transferer is a rich person, so it requires the poverty measure to decrease if a transfer is made from a rich person to a poor one. *The strong upward transfer axiom* requires the poverty measure to increase when a transfer is made from a poor person to a richer poor person which makes the later rich. *The strong downward transfer axiom* requires the poverty measure to decrease when a transfer is made from a rich person to a poor one which makes the later rich. *The continuity axiom* requires the poverty measure to be continuous in income. They also consider two definitions of the poor.
By examining the interrelationships among various combinations of the axioms, Donaldson and Weymark found that for each combination of the axioms, one of three cases can occur:

1. The axioms maybe inconsistent.
2. Considering specific axioms combination implies a stronger version of one or more of these axioms.
3. We can find a poverty index that satisfies this set of axioms and any weaker versions of the monotonicity and transfer axioms implied by this set.

The first two cases are verified by introducing eleven theorems some are impossibility theorems which verify case one and the rest are concerned with the strong versions of the axioms that have been satisfied in the axiom combinations they consider which verify case two. Note that all the impossibility theorems occur using the strong definition of the poor, while the rest are not. Moreover, Donaldson and Weymark's theorems are based on the satisfaction of the focus axiom, so we might question the validity of these impossibility theorems without the satisfaction of the focus axiom? This question arises because some people do not believe that the focus axiom is essential for a poverty measure and that a better representation of the notion of "relative deprivation" violates the satisfaction of this axiom. The third case is verified by providing us with twelve new poverty indices all of which are taken from the already existing ones either by changing the weighting scheme associated with them or by adding (or subtracting) a positive number to the individual poverty gaps.
Donaldson and Weymark provided us with many axioms for the poverty measure. Too many to require for one poverty measure to satisfy simultaneously. However, you can look at only the axioms you are interested in -depending on the purpose you want for the poverty measure- and see the interrelationships among them which are provided in their paper.

Atkinson (1987):

Atkinson reexamined three basic issues in measuring poverty: the choice of poverty line,[34] the poverty index, and the relationship between poverty and inequality. In the choice of poverty index, Atkinson proposed a class of poverty measures satisfying general properties and when extra two conditions are satisfied, all members of that class give the same ranking.
Atkinson proposed the following class of poverty measure:

$$G(P) = \int_0^A P(y, \pi) f(y) dy, \tag{8}$$

where $P(y, \pi) = 0$ for $y \geq \pi$ and G is a decreasing function.
and f(y)is the density function corresponding to y.
Atkinson defined P as a class of additively separable poverty measures and $G(P)$ is a monotonic transformation of P which can be written as an integral of the function $P(y, \pi)$ over the whole income distribution (i. e., from 0 to A). Atkinson expressed poverty negatively so that G is a decreasing function in P. Moreover, he assumed P is nondecreasing in y so that $P(y, \pi)$ is nonpositive.
Atkinson wanted all members of his class of poverty measure to give the same ranking so that we can say that poverty has increased or decreased for all poverty measures in the class. To do this, Atkinson proposed two conditions:
Condition I: A necessary and sufficient condition for there to be for all $\pi \in \pi^*$ a reduction or no increase in poverty for all measures in the class given by equation (8), where P is continuous and nondecreasing in Y, on moving from the distribution F^1 to F is that:

$$\Delta F(\pi) \leq 0 \qquad \forall \pi \in [0, \pi^+],$$

where:
$\pi^* = [\pi^-, \pi^+]$ is the range over which the poverty line π can vary.
$F(y)$ is the cumulative distribution function.
F, F^1 are two distributions.
$\Delta F = F - F^1$.

This condition is concerned with poverty measures that are continuous and nondecreasing in y from the class of poverty measures introduced in equation (8). In this case, poverty will not increase if the difference in the cumulative distribution function of two income distributions is nonpositive (which is equivalent to saying that the difference in poverty given by $G(P)$ in nonnegative).
Rodgers (1991) commented on this condition as of limited usefulness.[35]
Condition II: A necessary and sufficient condition for there to be for all $\pi \in \pi^*$ a reduction, or no increase, in poverty for all measures in the class given by equation (8), where P is continuous, nondecreasing, and (weakly) concave in Y, on moving from the distribution F^1 to F is that:

$$\Delta\phi(\pi) = \int_0^{\pi} \Delta F(Y)dy \leq 0 \quad \forall \quad \pi \in [0, \pi^+],$$

where: $\phi(\pi)$ is the poverty deficit curve.
This condition adds another assumption to $P(y, \pi)$ which is (weakly) concave in Y, i. e. P is differentiable and $P_{yy} \leq 0$.

Atkinson noted that condition II is implied by, but does not imply, condition I so that the second condition is weaker. Moreover, the second condition is not strong enough to ensure the same ranking by the head-count ratio. To make the second condition sufficient, Atkinson suggested that we have to assume P is concave in income; in other words, that it satisfies the Dalton transfer principle. By assuming this condition to hold, the result can be strengthened to include measures that are s-concave but not necessarily additively separable like Thon measure of 1979.

Atkinson has achieved his goal in reaching a degree of agreement even when judgments differ. He provided us with a class of poverty measures that all its members give the same result when applied to a certain situation, while other measures may give contradicting results when applied to the same situation.

Shorrocks (1995):

Shorrocks criticized Sen's poverty measure for -besides not being subgroup consistent- having three disadvantages: It does not satisfy the transfer axiom, it is not a continuous function of income, and it is not "replication invariant" meaning its value does not vary with replication. Shorrocks, however, responded -as Sen did- to the third disadvantage that he mentioned by stating the most commonly used version of Sen's measure:

$$P_s(y, \pi) = H[I+G(1-I)].$$

Shorrocks said that the other two disadvantages come from the normalization factor used by Sen; he criticized Sen's normalization factor for being "too demanding".[36] Therefore by changing it we can get an index that satisfies the transfer axiom and is continuous in individual income *but* it still will not be subgroup consistent.
Shorrocks modified the normalization factors used by Sen to give us the following index:

$$P_{SH}(y, \pi) = (2 - H)HI + H^2(1 - I)G. \qquad (9)$$

Shorrocks mentioned that this index -in spite of avoiding the two disadvantages of Sen's measure- should not be considered as one of the standard poverty measures. It is similar to that of Takayama (1979) and Thon (1979).[38]
Shorrocks mentioned that the major advantage of his index is the ability to be interpreted with the poverty gap profile.[39] He interpreted his index as the ratio of the area under the poverty gap profile to the area under the line of maximum poverty (which is obtained if all incomes are zero).

The Second Group:

This group includes but is not limited to Takayama (1979), Blackorby and Donaldson (1980), Clark et al (1981), and Hagenaars (1987).

Takayama (1979):

Takayama proposed an alternative poverty measure. He introduced the "censored income distribution" $(y^*(\pi))$ truncated from above by the poverty line, which he defined as:[41]

$$y_i^*(\pi) = y_i \quad if \quad y_i < \pi,$$

and

$$y_i^*(\pi) = \pi \quad if \quad y_i \geq \pi.$$

He then applied the Gini measure of inequality (G) to this censored distribution to get a poverty measure as:

$$P_{tk} = H[(1 - \phi)I + \phi G_w], \qquad (10)$$

where ϕ is the cumulative income ratio of the poor, defined as:
$\phi = H\ \bar{y}_p / \bar{y}_c$, and where $\bar{y}_c$ is the mean income of the censored income distribution and G_w is the Gini coefficient within the poor.

In this way, Takayama, like Sen, can write his measure as a function of the head count ratio, the income gap ratio, and the Gini measure of inequality among the poor. However, the interpretation differs a little between them. Takayama interpreted his measure as the normalized weighted *average* of the poor persons' gaps since the weights, (1 - ϕ) and ϕ, add up to one, while Sen's measure can be interpreted as the normalized weighted *sum* of the poor persons' gaps since the weights, 1 and (1 - *I*), do not necessarily add up to one.

Takayama claimed that his measure is better than Sen's measure in some respects. On the other hand his measure does not satisfy Sen's monotonicity axiom. Takayama's claim has been a subject for criticism after that.[44]

Blackorby and Donaldson (1980):

Blackorby and Donaldson introduced two kinds of poverty measures: The first one is *the relative poverty measure* which is homogeneous of degree zero[45] in all incomes and the poverty line. They arrived at this measure by replacing the Gini inequality measure used by Sen with another form of relative inequality measure. The second one is *the absolute poverty measure* which is homogeneous of degree one[46] in all incomes and the poverty line. To get this kind of measure you have to use an absolute measure of inequality.

Blackorby and Donaldson used the notion of representative income which, like equally distributed equivalent income, means the level of income that if given to each poor person would result in the same welfare level as the current income distribution. They showed that for every homothetic social evaluation function, there is one relative poverty measure and that Sen's measure is a relative measure that uses the Gini social evaluation function. For the absolute poverty measure, they showed that for every translatable social evaluation function there is one absolute poverty measure.

Relative Poverty Measure:

Blackorby and Donaldson suggested that they can get different poverty measures just by using different relative inequality measures. Doing this, they obtained the following measure:

$$P_{bd}^{r}(y,\pi) = (q/n)[(\pi - \xi^{p})/\pi] \qquad (11)$$

where ξ^{p} is the representative income of the poor as measured by any homothetic social evaluation function.

From equation (11), we can see that we can get a different relative poverty measure for every homothetic social evaluation function.

Absolute Poverty Measure:

For policy purposes, Blackorby and Donaldson thought it might be helpful to have a poverty measure based on the absolute shortfall of the representative income of the poor from the poverty line and the absolute number of the poor, so they introduced this absolute poverty measure:

$$P_{bd}^{r}(y,\pi) = q(y,\pi)[\pi - \xi^{p}], \qquad (12)$$

from equation (12), we can see that we can get a different absolute poverty measure for every translatable social evaluation function defined on the incomes of the poor. Note that when all the poor have the same income, P_{bd}^{r} is equivalent to the head count ratio multiplied by the representative income gap ratio for the poor, i. e., *HI*, while P_{bd}^{r} is equivalent to the aggregate poverty gap.

Foster (1984) argued that you can look at Blackorby and Donaldson's measures as too small in one sense and too large in another sense. In addition, while these measures satisfy Sen's focus and weak transfer axioms, there is no guarantee that they will satisfy the monotonicity axiom. Foster (1984) suggested that to make sure these measures satisfy the monotonicity axiom, the underlying "representative income" or the "social evaluation function" must be *strictly increasing* in the incomes of all the poor.

Clark, Hemming, and Ulph (1981):

Clark et al. introduced two poverty measures, each one is supposed to overcome one weakness of Sen's measure. *The first weakness* of Sen's measure they criticized is the transfer sensitivity which was recognized by Kakwani (1980). Clark et al. thought that the distribution of income among the poor is usually most densely populated at the top of it (i. e., near the poverty line) and there where transfer sensitivity will be at its greatest, but following Kakwani, transfer sensitivity should be greater at the bottom than at the top of the distribution. *The second weakness* of Sen's measure they criticized is the way Sen used the concept of relative deprivation which was recognized by Takayama (1979). Since Takayama's measure does not satisfy the monotonicity axiom, Clark et al. modified his approach to achieve monotonicity in the measure they provided.

THE FIRST MEASURE:

Clark et al. realized that Kakwani's problem arises because Gini measure is not a suitable measure of inequality to capture transfer sensitivity, so they used Atkinson measure of inequality. They expressed their measure by using inequality measure defined over poverty gaps instead of incomes.

Clark et al. applied the negative of Atkinson inequality measure to the vector of poverty gaps to give:[48]

$$B_g^{\propto} = \frac{([1/q] \sum_{i \in T} g_i^{\propto})^{[1/\propto]}}{\bar{g}} - 1$$

where $\bar{g} = (1/q) \sum_{i \in T} g_i$ is the mean poverty gap. Using this, Clark et al introduced their first poverty measure:

$$P_c(y, \pi) = HI(1 + B_g^{\propto}) \qquad (14)$$

Clark et al. mentioned that this measure is increasing in *H, I,* and $\propto$. Also they noted that when $\propto = 1 \Rightarrow P_c(y, \pi) = HI$ and as $\propto$ gets larger $P_c(y, \pi)$ becomes "more sensitive to all transfers". Most importantly, their measure satisfies the monotonicity axiom in addition to the other Sen axioms.

Foster (1984) found that if Blackorby and Donaldson use $B^{\propto}$ for their poverty measure, then the resulting measure may not satisfy the monotonicity axiom. Foster also mentioned that if Atkinson's inequality measure $A^{\propto}$ is used as a measure of inequality instead of $B^{\propto}$, then the Blackorby and Donaldson's method will satisfy the monotonicity axiom, while the Clark et al.'s method may not.

The Second Measure:

In the second poverty measure, Clark et al. modified Takayama's poverty measure to obtain one that satisfies the monotonicity axiom. To do so, Clark et al. used the same concept of "censored distribution" $y^*(\pi)$ developed by Takayama and applied the concept of "representative income" to it. The second poverty measure they introduced is:

$$P_c^{\alpha}(y, \pi) = \{\pi - \bar{y}^* [1 - A^{\alpha}(y^*)]\}/\pi, \tag{15}$$

for $\alpha < 1$,

where $\bar{y}^* = (1/n) \sum_{i \in N} y_i^*$ is the mean of the censored distribution,

and $A^{\alpha}(y^*)$ is the Atkinson measure of inequality applied to the censored distribution.

As Foster (1984) interpreted it, this measure can be looked at as another income gap ratio but using a different income distribution which is the representative income of the censored income distribution. The advantage of this measure is that it satisfies the monotonicity axiom in addition to the rest of Sen's axioms.[49]

Hagenaars (1987):

Hagenaars examined the already existing axioms that have been introduced in the literature and picked seven of them as requirement for the poverty measure. He checked nine of the already existing poverty measures and found none of them was satisfying all the required axioms simultaneously, so he introduced his own class of poverty indices. Hagenaars used different income inequality measure to drive his class of poverty indices. He applied the income inequality measures of Dalton (1920) and Atkinson (1970) to the poverty measurement. Hagenaars used a utilitarian social welfare function and showed that a generalization of it is straightforward; he illustrated that by considering a Gini-type social welfare function. The satisfaction of the required axioms are translated into requirements for the social welfare function he used.

The required axioms for Hagenaars are: focus axiom, monotonicity axiom, transfer axiom transfer sensitivity axiom, decomposability axiom,[50] population symmetry axiom, and proportion of poor axiom.

All these axioms have been explained before except the last two:

Population Symmetry Axiom: If two or more identical populations are pooled, the poverty measure should not change.

This axiom is well-known in income inequality literature not in poverty. It holds for the head-count ratio, the relative income gap, and all well-defined income inequality measures. Therefore, if we combine these three elements into a poverty measure, then this poverty measure will have this property as well.

Proportion of Poor Axiom: An increase in the relative number of the poor should increase the poverty measure.

For this axiom, Hagenaars distinguished between two cases when the size of total population is constant and when it varies.

Hagenaars applied the social welfare function SW to the censored income distribution y* to get:

$$SW(y^*) = (1/n) \sum_{K=1}^{n} \min\ [U(y_k), U(\pi)],$$

where U is the utility function.
Then, he used the Dalton inequality measure[52] to get the following poverty measure:

$$PHD - 1 \ \underline{SWU(irY;)} \sim;7\sim[>\sim_1 U(j;(;(\sim i)_1 \quad (16)$$

which equals the head-count ratio times the relative average welfare gap of the poor.
Using Atkinson inequality measure, Hagenaars obtained the following poverty measure:

$$P_{H,A} = 1 - \frac{y^*_{EDE}}{\pi} \quad (17)$$

where $y^*_{EDE} = U^{-1}[SW(y^*)] = u^{-1}\{(1/n)[\sum_{i=1}^{q} U(y_i) + (n - q)U(\pi)]\}$.

Hagenaars found that the necessary and sufficient conditions for the proposed poverty measures to satisfy all the required axioms[53] -except for one case of the proportion of poor axiom[54] - are that U is a strictly concave, continuous, and increasing function. For that case of the proportion of poor axiom to be satisfied, we have to have $U = 0 \ \forall \ y_i < \pi$ and in this case $P_{H,A}$ is not defined.
Hagenaars' poverty measures are derived by assuming that the poverty line is set less than the average income in society, i. e., $\pi < \bar{y}$. In the opposite case,[55] i. e., $\pi > \bar{y}$, Hagenaars found that the poverty measures in this case will asymptotically be the Dalton and Atkinson measures of inequality themselves.[56]

By using the Gini-type social welfare function, e. g. $SW(y) = \sum_{i=1}^{n} w_i y_i$, Hagenaars obtained another

poverty measure:

$$P_{H,W} = 1 - \frac{\sum_{i=1}^{n} w_i y_i^*}{\pi \sum_{i=1}^{n} w_i}$$

Note that $P_{H,W}$ is derived using the Dalton measure of inequality also some restrictions are imposed on the weights w_i in order for $P_{H,W}$ to satisfy the required axioms.
Hagenaars noticed that his general class of poverty indices can be expanded in many ways by allowing different functional forms of U.[58]

3.2.3 The Third Group:

This group includes but is not limited to Foster et al. (1984), Foster and Shorrocks (1991), Rodgers and Rodgers (1991), and Wright (1996).

3.2.3.1 Foster, Greer, and Thorbecke (1984):

Foster et al. looked at different dimension of a poverty measure. They wanted the poverty measure to be decomposable into subgroups defined on different criteria like ethic or geographical lines or others.

The class of poverty measures they introduced is:

$$P_\alpha(y, \pi) = (1/n) \sum_{i \in T} (g_i/\pi)^\alpha, \quad (18)$$

when $\propto = 0 \Rightarrow P_0(y, \pi) = H$ and when $\propto = 1 \Rightarrow P_1(y, \pi) = - HI$.

The case Foster and colleagues are interested in is when $\propto = 2$. Note that a can be interpreted as a measure of "poverty aversion"; a larger α gives more emphasis to the poorest poor individual.

$P_\propto(y, \pi)$ can be interpreted, as Sen did, as a weighted sum of the income gaps of the poor where the weights are given by the gaps themselves (not the rank order as the case with Sen). Note that these weights are related to the notion of relative deprivation[59] as Sen required. Furthermore, Foster et al. showed that $P_2(y, \pi)$ is related to a well-known measure of inequality which is the squared coefficient of variation c^2 in the same way $P_s(y, \pi)$ is related to the Gini coefficient. To see this,[60] equation (18) can be written as:

$$P_2(y, \pi) = H[I^2 + (1 - I)^2 c_p^2]. \qquad (19)$$

Note that $P_\propto(y, \pi)$ satisfies the monotonicity axiom for $\propto > 0$, the transfer axiom and weak transfer axiom for $\propto > 1$, and the transfer sensitivity axiom for $\propto > 2$.

All of the previous properties were not the main objective of Foster and colleagues in constructing the poverty measure, they wanted it to be decomposable into subgroups which it did.

$P_\propto(y, \pi)$ also satisfies the subgroup monotonicity axiom proposed by Foster et al. which says:

Subgroup Monotonicity Axiom: Let y' be a vector of incomes obtained from y by changing the incomes in subgroup j from $y^{(j)}$ to $y'^{(j)}$ ceteris paribus. If $y'^{(j)}$ has more poverty than $y^{(j)}$, then y' must also have a higher level of poverty than y.

This axiom says that if the income of a subgroup of the population changes, then the total poverty should move in the same direction with the subgroup, ceteris paribus.

Foster et al. provided us with a poverty measure that satisfies the decomposability property and the subgroup monotonicity axiom in addition to all of Sen's axioms. This may not be the only measure that does so but it is the first one introduced in the literature concerning this idea.

Foster and Shorrocks (1991):

Foster and Shorrocks introduced a class of poverty indices that satisfy the *subgroup consistency axiom* which is a stronger version of the *subgroup monotonicity axiom*. The subgroup consistency axiom requires the overall level of poverty to increase if poverty increases within a subgroup of the population and remains the same outside that subgroup; in this axiom *the poverty value* has to remain the same outside that subgroup, while in the subgroup monotonicity axiom *the income distribution* is the one that has to remain the same outside that subgroup, so the former is a stronger version of the later.

Foster and Shorrocks imposed two restrictions on the subgroup consistency axiom. *The first one* required the subgroup sizes to be fixed to exclude changes in subgroup poverty due to population shifts. *The second one* required the number of subgroups to be two[62] in which the level of poverty increases and unchanges in the two subgroups respectively. However, they noticed that replacing unchanged with not decreasing in the second subgroup will not affect the axiom.

Foster and Shorrocks characterized the class of subgroup consistent poverty indices and identified the special features associated with this property. To do that, they kept the other properties of poverty indices to the minimum: they assumed that their class of poverty indices satisfy these five properties: focus, monotonicity, symmetry, replication invariance, and restricted continuity. *The symmetry axiom* allows incomes to be re-ordered without affecting the poverty value. *The replication invariance axiom* ensures that the index treats poverty in per capita terms. *The restricted continuity axiom* restricts the index to be continuous in the incomes of the poor only. Using continuity[63] as a property associated with subgroup consistency, Foster and Shorrocks provided us with the first poverty index, which is continuous and subgroup consistent:

$$P_{fs}(y, \pi) = F\{1/[n(y)] \sum_{i=1}^{n(y)} \phi(y_i)\} \quad \forall \; y \in Y, \qquad (20)$$

where $\phi: y \rightarrow R$ is a continuous and nonincreasing function;[64] and $\phi(t)=0 \;\; \forall \; t \geq \pi$

and $F: \phi(y) \rightarrow R$ is a continuous and increasing function.

They defined a *canonical index,* P^{ϕ}, as:

$$P^{\phi}(y, \pi) = 1/n(y) \sum_{i=1}^{n(y)} \phi(y_i) \quad \forall \; y \in Y,$$

Foster and Shorrocks interpreted the canonical index P^{ϕ} as the average deprivation in the whole society, then equation (20) tells us that any continuous subgroup consistent poverty index must be continuous and increasing transformation of a canonical index.

Moreover, Foster and Shorrocks gave us two more results to show that for each continuous subgroup consistent index there is a decomposable index which ranks distributions in exactly the same way. Even when the continuity assumption is dropped, Foster and Shorrocks gave us two more results similar to the ones of continuity assumption but with lexical forms of indices substituting for continuity.[66] The above indices assumed that the poverty line is fixed. Allowing variations in the poverty line, Foster and Shorrocks distinguished between two poverty indices: relative poverty index and absolute poverty index.

To avoid any criticism of their poverty indices satisfying only some axioms, Foster and Shorrocks said that the additive form of the canonical index makes it easy to incorporate other axioms as well.

Rodgers and Rodgers (1991):

Rodgers and Rodgers looked at subgroup population from a different prospective. Instead of proposing a measure for the amount of poverty in the subgroup, they proposed a measure for the intensity of poverty within the subgroup relative to that of the population as a whole. Rodgers and Rodgers suggested that their poverty intensity index be used as a component, not a substitute, for the standard poverty indices.

Rodgers and Rodgers wanted the proposed index to satisfy the following axioms: *focus, anonymity* which introduced by Hagenaars (1987), *monotonicity (M1)* which is the same as the one introduced by Sen (1976), *monotonicity (M2)* which is the same as the strong monotonicity axiom introduced by Donaldson and Weymark (1986), *transfer (T1)* which is the same as the weak transfer axiom introduced by Sen (1977), *transfer (T2)* which is the same as the strong upward transfer introduced by Donaldson and Weymark (1986), *monotonicity sensitivity, transfer sensitivity I, transfer sensitivity II,* and *additive decomposability.*

Rodgers and Rodgers defined the poverty intensity index *(PI)* in sub-population k as:

PI_k = (the proportion of population poverty contributed by group k)/(the proportion of population size contributed by group k)

$= (\pi_k)/[n_k/n],$

and π_k defined as:
π_k = (contribution by group k to the population poverty index value)/(the poverty index value for the whole population).
This approach can be applied to all poverty indices already existing. In the case of additively decomposable poverty indices,[68] Rodgers and Rodgers found that the poverty intensity of group k is the poverty index of group k divided by the poverty index of the entire population, i. e.,

$$PI^{+}_{k} = (P^{+}_{k})/(P^{+}), \qquad (21)$$

where $P+$ is any additively decomposable poverty index.

However, it is not that straightforward for nondecomposable index.[69] In this case, the poverty intensity in group k can be written as:

$$PI^{A}_{k} = \frac{\sum_{j \in q(k)} g_j w_j / \sum_{i=1}^{q} g_i w_i}{n_k/n} \qquad (22)$$

where $q(k)$ is the set of poor people in group k, and equal gaps receive equal weights (i. e., $W_n = W_j$ if $g_n = g_j$).
Rodgers and Rodgers noted that in equation (22), each gap in the numerator has the same weight as that gap in the denominator[70] and that this weight does not change with the way the population is partitioned.

By applying their measure to the data, Rodgers and Rodgers found that in the case of additively decomposable poverty indices they considered - which includes Foster et al. index, H, and HI - both the poverty indices and poverty intensity indices give the same poverty ranking of "any set of disjoint groups" of a given population. However, this result does not hold for the nondecomposable index they considered which is Sen's index. Therefore, they strongly believed that theory and practice support the use of PI_k^{FGT} as a measure of poverty intensity.[71]

Wright (1996):

Wright standardized Foster et al. (1984) poverty measure to control[72] for the effect of "compositional factors" on the measurement of poverty. The standardized poverty measure he provided also satisfies Sen's axioms.
Wright rewrote Foster et al.'s measure as:

$$P(\alpha) = \sum_{k=1}^{K} s_k P(\alpha)_k ,$$

where k = 1,2...., K are mutually exclusive and exhaustive subgroups of the population.
And s_k is subgroup K's relative population share[73] i. e., $s_k = n_k/N$.
Wright interpreted $P(\alpha)$ as the "total" poverty rate for the whole population. He then introduced the subgroup's share of this total poverty as:

$$S(\alpha)_k = [s_k . P(\alpha)_k]/P(\alpha).$$

Wright applied shift-share analysis[74] to the Foster et al.'s measure to get:

KKAP(a) - ~ Σ ASk P̄(a)k + ~ ~skAP(a)k, (23)
k=1 k=1

where $\Delta P(\alpha)_k = P(\alpha)_{kt} - P(\alpha)_{k(t-1)}$,
$\Delta s_k = s_{kt} - s_{k(t-1)}$,
and a bar above the variable gives a linear combination of the values of this variable in the base period and the final period.

Equation (23) decomposes the change in the total poverty rate [$\Delta P(\alpha)$] into two parts: The first part is due to changes in the population's shares or composition (Δs_k) and the second part is due to changes in the poverty rate for each subgroup of the population [$\Delta P(\alpha)_k$].

Calculating the decomposition and generalizing it, Wright wrote the standardized version of Foster et al. measure as:

$$P(\alpha)^*_t = \sum_{k=1}^{K} s_k^* . P(\alpha)_{kt}, \qquad (24)$$

where s_k^* is subgroup k's relative population shares from a "standard population"[75].

Wright reminded us that, despite the fact that this standardized version of Foster et al. measure is independent of changes in compositional factors and satisfies Sen axioms, it has no direct meaning which, he said, is common for all directly standardized rates.

Wright also used data from the Family Expenditure Survey to examine the absolute and relative poverty in the U.K. and to compare the unstandardized with the standardized poverty rates. He found that the estimated poverty rate is different using standardized vs. unstandardized poverty measures. His finding was true for both the absolute and the relative poverty he estimated.

After looking at all these measures one might wonder if every measure that was introduced after Sen's measure did make a difference in measuring the level of poverty in any society? To answer this question, the author is going to mention what Rodgers (1991) found in his study. Rodgers (1991) used ten[78] poverty measures to rank poverty among states using the 1980 United States Census data.

Rodgers found that the choice of the poverty index to use in reality matters only if we are choosing between one of the traditional measures and one of the new[79] poverty measures. In comparing among all new poverty indices, Rodgers found that the ranking of poverty giving to the states by using the new poverty measures were almost the same. Moreover, Rodgers found that there is a high degree of correlation among new poverty measures; he found that Sen, Takayama, Thon, Kakwani, Blackorby and Donaldson, and the normalized deficit

(HI) ranked 21 out of 35 states -that he included in his research- identically and that Clark et al. and Foster et al. ranked 23 out of 35 states identically.
At the end, all these measures still are not sensitive to the neighborhood in which the family lives.

THE NEIGHBORHOOD ADJUSTED POVERTY MEASURE:

In 1994, Abdel-Raouf criticized the existing poverty measure for being concerned with measuring the monetary aspects of poverty only, in other words for using income as the only indicator of poverty. She included the neighborhood quality in her measure of poverty since it has proven to affect the life chances of the individuals living in it.

To reflect the neighborhood quality in the poverty measure, Abdel-Raouf introduced a new concept of income which is the "Neighborhood Adjusted Income (NAI)".[81] The neighborhood adjusted income *(NAI)* is a function of the monetary income the person receives (y_i), the neighborhood characteristics in which the person lives (n_{ij}) and how these neighborhood characteristics affect the individual living in it (S_{ij}). To put it in mathematical terms:

$$NAI_i = y_i + \sum_j S_{ij}(n_{ij} - \bar{n}_j)c_jR \qquad (25)$$

where y_i is the monetary income that person i receives

n_{ij} is the actual percentage of the neighborhood characteristic j in which person i lives

$\bar{n}_j$ is the average percentage of the neighborhood characteristic j

c_j is the increase in the number of years in school that is attributed to a one percent increase in the neighborhood characteristic j

R is the return to a one year of school.

and

$$S_{ij} = \begin{cases} +1 & \text{if the neighborhood characteristic j has a positive effect on person i.} \\ 0 & \text{if the neighborhood characteristic j has no effect on person i.} \\ -1 & \text{if the neighborhood characteristic j has a negative effect on person i.} \end{cases}$$

The neighborhood characteristic j has a positive effect on person i if the derivative of the i^{th} person utility function[82] with respect to neighborhood characteristic j is positive, the neighborhood characteristic has no effect if the derivative is zero and the neighborhood characteristic has a negative effect if the derivative is negative.[83]

To account for the fact that c_j does not need to be the same for everyone in the neighborhood, Abdel-Raouf added a random error to c_j to become $(c_j + \xi_{ij})$. Substituting this term in equation (25) yields:

$$NAI_i = y_i + \sum_j S_{ij}(n_{ij} - \bar{n}_j)(c_j + \xi_{ij})R. \qquad (26)$$

Abdel-Raouf mentioned that we can take care of the effect of the neighborhood

in any of the existing poverty measure by using the neighborhood adjusted income (NAI) instead of the monetary income.
The poverty measure Abdel-Raouf introduced was obtained from Sen's measure by replacing the monetary income used by Sen with the neighborhood adjusted income to get:

$$P_{FR}(NAI, \pi) = \frac{2}{(q+1)n\pi} \sum_{i \in T} h_i r_i \quad (27)$$

where q is the number of poor in the population[84]
n is the population size
π is the poverty line
h_i is person's i poverty gap which is defined as $h_i = \pi - NAI_i$
r_i is the rank given to person's i poverty gap
and NAI is the neighborhood adjusted income for the whole community.

Note that r_i is defined the same way as Sen did, with the difference due to different poverty gaps. So $r_i > r_j$ if $h_i > h_j$.

This poverty measure can be interpreted as a normalized weighted summation of the individual poverty gaps using a modification of the income of the poor in such a way that takes into account the advantages or disadvantages they get from their neighborhoods.[85] This measure satisfies the focus axiom, the monotonicity and the weak transfer axiom.[86]

Then Abdel-Raouf compared this measure with Sen's measure by applying them to the data. The data used is a .05% random sample of the 1980 Public Use of Microdata Samples (PUMS) -special tabulation. The result supported her idea by having a higher level of poverty using her proposed measure than by using Sen's measure since most of the poor in the sample were living in poor neighborhoods.

CONCLUSION

In this paper, the author introduced the reader to the poverty measures available in the literature which are divided into three groups. The first group is the traditional poverty measures which -despite the criticism that they had since 1976- are the ones used in the official estimation of poverty. This group is divided into two types: The first one is based on counting the poor in the population and the second one is based on the gap between income and the poverty line for poor people.

The second group is Sen's measure and the similar subsequent literature that followed Sen. In 1976, Sen introduced a new approach for the poverty measure to make it sensitive to the income level and distribution of the poor. Following Sen, huge literature has been introduced which is divided into three types: The first one varies the weights and/or the normalization factor of Sen's measure to get another poverty measure, the second one replaces the Gini measure of inequality used by Sen with another inequality measure to get different poverty measure, and the third one is concerned with poverty in a subgroup of the population and how it contributes to the total poverty.

The third group is the neighborhood adjusted poverty measure. In 1994, Abdel-Raouf introduced a new approach for the poverty measure to make it -in addition to being sensitive to the income level and distribution of the poor- sensitive to the neighborhood characteristics in which the family lives.

This paper provides a comprehensive and objective survey of aggregate poverty measures introduced in the literature. To avoid being too long, some of the details have been omitted, however, the reader was referred to the source needed in each case[89]. Subjective opinions were limited in order to provide the reader with a fair survey of the literature so that the reader can build his/her own opinion.

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ENDNOTES

[1]For example, District of Columbia (22%) and New Mexico (20.8%). U.S. Census Bureau, Poverty 1998.
[2]See U.S. Census Bureau, statistical brief about the poverty areas.
[3]See, for example, Abdel-Raouf (1994), Crane (1991), and Massey, Gross, and Eggers (1991).
[5]I am combining Sen's work from 1976 and later since the focus axiom was implicitly introduced in 1976 but explicitly after that. Also the weak transfer axiom was introduced first as the transfer axiom, which differs from the first in that in the later one the number of the poor can change (by allowing the recipient of the transfer to cross the poverty line), but in the first the number of the poor has to stay the same.
[6]This argument was presented by Sen in 1981, pp.31-32.
[9]Sen noted that for a large number of the poor, q is large enough so that $q/(q+1) \approx 1$, $P_s(y, \pi)$ can be written as(which is the most commonly used form of Sen's measure):

$$P_s(y, \pi) = H[I + G(1 - I)].$$

[10]By that expression, Anand meant the level of income per person which, if equally distributed, would give the same level of social welfare as the existing income distribution.
[12]Anand mentioned that if all the poor have the same income, then P_a is equivalent to the aggregate poverty gap as a fraction of the total incomes while P'_a is equivalent to the aggregate poverty gap as a fraction of the total nonpoor incomes.

[13]I am going to mention only the strong-form requirements; the other three requirements are just the weak forms of the ones mentioned.

[14]Which he called equalizing transfer.

[15]Which he called disequalizing transfer.

[16]This is different from Sen's ranking system for the poor among the poor only.

[17]This interpretation of the weights is taken from Foster (1984).

[18]Which is the population monotonicity axioms they introduced, see page 8.

[19]Note that Kakwani sees these axioms as important for any poverty measure to satisfy in order to be sensitive to the income inequality among the poor in addition to Sen's monotonicity and transfer axioms.

[20]Note that Kakwani argued that the weighting scheme used by Sen is opposite to what it is supposed to be-which is that the weight has to be largest when the transfer is from the poorest person, then it goes down with an increase in the level of income of the transferer. However, I cannot see this and I think that Sen's weighting scheme is completely correct.

[21] For proof, see Foster 1984.

[22]Kundu and Smith did not provide us with a specific poverty measure to use if the population size is not fixed. However, they introduced the axioms for a measure to satisfy in this case.

[24]A transfer that keeps the order of incomes unchanged.

[25]The last two axioms are the "population monotonicity" axioms mentioned above.

[26]From the class of poverty measures they adopted, which was defined above.

[27]Monotonically decreasing means if x and y are two income distributions then $x < y \Rightarrow P(x,\pi) > P(y,\pi)$.

[28] See Kunda and Smith (1983) for their justification of this contradiction.

[29]See Sen 1981 and Foster 1984 for details.

[31]They noted that Thon's measure of 1979 satisfies all of their axioms if the weak definition of the poor is used, when the strong definition of the poor is used, the upward monotonicity and strong downward transfer axioms are not satisfied.

[34]The choice of poverty line is not of our interest at this point.

[35]Rodgers (1991) gave two reasons for his claim, See Rodgers (1991) for details.

[36]Shorrocks criticized Sen for allowing the normalization factor to depend on the poverty line and all incomes as this may change the rank order. Rather, he allowed the normalization factor to depend only on n, q, z, and the mean income of the poor.

[38]Shorrocks sees that his measure is superior to both since Takayama's measure is not monotonic and Thon's measure is not replication invariant.

[39]Shorrocks believes that the poverty gap profile will be very important tool in poverty analysis and it will do the same job as Lorenz curve in inequality analysis.

[41]Note that Foster (1984) is mistaken in his representation of Takayama's "censored distribution" . Foster represented it as (p.235)

$$y_i^{*}(\pi) = y_i \ \forall \ i \in T(y, \pi) \text{ and } y_i^{*}(\pi) = \pi \ \forall i \notin T(y, \pi),$$

so the person with income equal to π in the old distribution will get $y_i^{*}(\pi) = y_i$ according to Foster since T includes all persons with income $\leq \pi$ but according to Takayama's definition he will get $y_i^{*}(\pi) = \pi$.

[44]See, for example, Foster 1984.

[45]In other words, doubling all incomes and the poverty line will not change the index.

[46]In other words, doubling all incomes and the poverty line *doubles* the index.

[48]The explanation of how to get to Clark et al. poverty measure is taken from Foster 1984.

[49]For proof, see Foster 1984.

[50]This axiom, as you will see in the third group, says that the poverty measure has to increase if a poverty in a subgroup of the population increases, ceteris paribus.

[52]For details on Dalton inequality measure, see Hagenaars 1987, p.590.

[53]To see the conditions required for satisfying each axiom alone, see Hagenaars 1987, p.591-592.

[54]It is the case when the size of the total population is constant and the increase in the relative number of the poor is due to the transfer of income, see Hagenaars (1987) for details.

[55]This case does not make sense to me since there is no meaning for the poverty line to be set greater than the mean income of the population.

[56]Hagenaars got that by using $SW(\bar{y})$ not $SW(\bar{\pi})$ as a measure of the situation of the poor. The disadvantage of using $SW(\bar{y})$ is that the resulting poverty measures do not satisfy the monotonicity nor the focus axioms.

[58] See Hagenaars 1987 for details, also see the special case that is easy to calculate which he provided using $U(y) = \ln y, y > 0$.

[59]To see how, see Foster et al. 1984 p.762.

[60]See Foster et al. (1984) for more details.

[62]They mentioned that this requirement is imposed only for simplicity, but the axiom will hold for any number of subgroups as well.

[63]Note that this continuity property is not restricted -which means that P is continuous in the whole income distribution- the restricted one is assumed in all the poverty indices they introduced.

[64]The function ϕ can be interpreted as a measure of deprivation which takes its minimum level 0 at π and keeps this value for all nonpoor.

[66]See Foster and Shorrocks 1991 p.698 - 700 for details.

[68]Rodgers and Rodgers used Foster et al. 1984 measure, the head count ratio (H), and the normalized deficit *(HI)* as examples of this case.

[69]Rodgers and Rodgers used Sen's poverty measure as an example of this case.

[70]See Rodgers and Rodgers 1991 for the two limitations they mentioned on that approach and how to avoid them.

[71]Except in one case, see Rodgers and Rodgers 1991 for details.

[72]Note that the method that Wright used for controlling for compositional factors is based on "shift-share analysis" and "direct standardization".

[73]Note that $\sum_{k=1}^{K} s_k = 1$.

[74]By using shift-share analysis, we can divide the change in the total rate into two parts each part has a useful explanation. See Wright 1996 for details.

[75]See Wright 1996 for explanation of standard population.

[78]These ten measures are Sen's, Takayama's, Thon's, Kakwani's, Blackorby and Donaldson's, Clark, Hemming and Ulph's, Foster, Greer and Thorbecke's, the normalized deficit *(HI)*, the head count ratio *(H)*, and the mean poverty gap ratio *(I)*.

[79]By new he meant the poverty measures introduced since Sen 1976 including Sen's measure.

[81]Note that NAI is higher than the monetary income the person receives if the person lives in a rich neighborhood and lower if the person lives in a poor neighborhood, while it stays the same if the person lives in an average neighborhood. So using the neighborhood adjusted income decreases the poverty status for the person in the first case increases it for the person in the second case and keeps it the same for the person in the last case.

[82]The utility function that Abdel-Raouf used takes the form:

$$U_i = t_j^{ai} + d_j^{ei} + w_j^{ri} - p_j^{Ii}$$

where t_j is the percentage of teenagers ages 14 to 18 in school in the neighborhood
d_j is the percentage of people with four or more years of college in the neighborhood
w_j is the percentage of whites in the neighborhood
p_j is the percentage of poor families in the neighborhood
a_i is the number of children for person i
e_i is the educational attainment for the householder of person i
r_i is the race for person i
and I_i is the income for person i

Note also that the generalization of this utility function to include other neighborhood characteristics is straightforward.

[83]The purpose of using S_{ij} is to reflect how the neighborhood characteristic j affects the individual i since the same neighborhood characteristic can affect different people in the neighborhood differently. To see that, consider the percentage of teenagers ages 14 to 18 in school in the neighborhood, for a family with children increasing this percentage will affect them positively but for a family without children in the same neighborhood, increasing this percentage will not affect them at all.

[84]Note that this number is different than the one used by Sen since the income distributions are different.

[85]See Abdel-Raouf (1997) for calculating this modification.

[86]See Abdel-Raouf (1997) for proof.

[89]The author stands ready to provide more details in any part of the paper upon request.

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